TOPIC 2

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use quadratic functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. According to the ___________, a product is 0 only if one (or more) of its factors is 0.
3. The ___________ of a quadratic function is $y = a(x - h)^2 + k$.
4. The ___________ of a quadratic function is the value of the radicand, $b^2 - 4ac$.
5. A number with both real and imaginary parts is called a ___________.
6. The ___________ of a quadratic function is $y = ax^2 + bx + c$.
7. ___________ is a method used to rewrite an equation as a perfect square trinomial equal to a constant.

- completing the square
- complex number
- discriminant
- imaginary number
- parabola
- quadratic function
- standard form
- vertex form
- Zero Product Property

Concepts & Skills Review

LESSON 2-1 Vertex Form of a Quadratic Function

Quick Review

The parent **quadratic function** is $f(x) = x^2$. The graph of the function is represented by a **parabola**. All quadratic functions are transformations of $f(x) = x^2$.

The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex of a parabola.

Example

What is the equation of a parabola with vertex (3, 1) and *y*-intercept 10?

$y = a(x - 3)^2 + 1$ ········ Substitute $(h, k) = (2, 3)$.

$10 = a(0 - 3)^2 + 1$ ········ Substitute *y*-intercept (0, 10).

$9 = a(-3)^2$ ········ Simplify.

$9 = 9a$

$a = 1$ ········ Solve for *a*.

$y = 1(x - 3)^2 + 1$ ········ Substitute *a*.

The equation of the parabola is $y = (x - 3)^2 + 1$.

Practice & Problem Solving

Describe the transformation of the parent function $f(x) = x^2$. Then graph the given function.

8. $g(x) = (x + 2)^2 - 4$
9. $h(x) = -2(x - 1)^2 + 5$

Identify the vertex, axis of symmetry, maximum or minimum, domain, and range of each function.

10. $g(x) = -(x + 3)^2 + 2$
11. $h(x) = 3(x - 4)^2 - 3$

Write the equation of each quadratic function in vertex form.

12. Vertex: (2, 1); Point (0, 4)
13. Vertex: (1, 5); Point (3, –1)
14. **Use Structure** The graph of the function $f(x) = x^2$ will be translated 4 units down and 2 units right. What is the resulting function $g(x)$?
15. **Make Sense and Persevere** Find three additional points on the parabola that has vertex (5, 3) and passes through (2, 21).

LESSON 2-2 Standard Form of a Quadratic Function

Quick Review

The **standard form of a quadratic function** is $y = ax^2 + bx + c$ where a, b, and c are real numbers, and $a \neq 0$. Use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex and the axis of symmetry. Substitue 0 for x to find the y-intercept of the quadratic function.

Example

The function $y = -8x^2 + 880x - 5{,}000$ can be used to predict the profits for a company that sells eBook readers for a certain price, x. What is the maximum profit the company can expect to earn?

The maximum value of a quadratic function occurs at the vertex of a parabola. Use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex.

$h = -\frac{880}{2(-8)}$	Substitute −8 for a and 880 for b.
$h = 55$	Simplify.
$x = 55$	Substitute h for x.
$y = -8(55)^2 + 880(55) - 5{,}000$	Substitute 55 for x.
$y = 19{,}200$	Simplify.

The vertex is (55, 19,200). The selling price of \$55 per item gives the maximum profit of \$19,200.

Practice & Problem Solving

Find the vertex and y-intercept of the quadratic function, and use them to graph the function.

16. $y = x^2 - 6x + 15$

17. $y = 4x^2 - 15x + 9$

Write an equation in standard form for the parabola that passes through the given points.

18. (1, 5), (3, 7), (6, 25)

19. (−2, 64), (3, −16), (7, 28)

20. **Higher Order Thinking** A golfer is on a hill that is 60 meters above the hole. The path of the ball can be modeled by the equation $y = -5x^2 + 40x + 60$, where x is the horizontal and y the vertical distance traveled by the ball in meters. How would you use the function to find the horizontal distance traveled by the ball and its maximum height?

21. **Make Sense and Persevere** The number of issues sold per month of a new magazine (in thousands) and its profit (in thousands of dollars) could be modeled by the function $y = -6x^2 + 36x + 50$. Determine the maximum profit.

LESSON 2-3 Factored Form of a Quadratic Function

Quick Review

Factor a quadratic equation by first setting the quadratic expression equal to 0. Then factor and use the **Zero Product Property** to solve. According to the Zero Product Property, if $ab = 0$, then $a = 0$ or $b = 0$ (or $a = 0$ and $b = 0$).

Example

Solve the equation $x^2 + x = 72$.

$x^2 + x - 72 = 0$	Set equation equal to 0.
$(x + 9)(x - 8)$	Factor.
$x + 9 = 0$ or $x - 8 = 0$	Zero Product Property.
$x = -9$ or $x = 8$	Solve.

The solutions for equation $x^2 + x = 72$ are $x = -9$ or $x = 8$.

Practice & Problem Solving

Solve each quadratic equation.

22. $x^2 - 6x - 27 = 0$

23. $x^2 = 7x - 10$

24. $4x^2 + 4x = 3$

25. $5x^2 - 19x = -12$

Identify the interval(s) on which each function is positive.

26. $y = x^2 - x - 30$

27. $y = x^2 + 11x + 28$

28. **Generalize** For what values of x is the expression $(x + 6)^2 > 0$?

29. **Model With Mathematics** A prairie dog burrow has openings to the surface which, if they were graphed, correspond to points (2.5, 0) and (8, 0). What equation models the burrow if, at its deepest, it passes through point (5, −15)?

LESSON 2-4 Complex Numbers and Operations

Quick Review

The **imaginary unit** i is the number whose square is equal to -1. An **imaginary number** bi is the product of any real number b and the imaginary unit i. A **complex number** is a number that may be written in the form $a + bi$. **Complex conjugates** are complex numbers with equivalent real parts and opposite imaginary parts.

Example

Write the product of $3.5i(4 - 6i)$ in the form $a + bi$.

$3.5i(4 - 6i)$

$= 3.5i(4) + 3.5i(-6i)$ .... Distribute.

$= 14i - 21i^2$ .... Simplify.

$= 14i - 21(-1)$ .... Substitute -1 for i.

$= 14i + 21$ .... Write in the form $a + bi$.

The product is $14i + 21$.

Practice & Problem Solving

Write each product in the form $a + bi$.

30. $(5 - 3i)(2 + i)$

31. $(-3 + 2i)(2 - 3i)$

Divide. Write the answer in the form $a + bi$.

32. $\dfrac{5}{3 + i}$

33. $\dfrac{2 - 3i}{1 + 2i}$

34. **Error Analysis** Describe and correct the error a student made when multiplying complex numbers.

$$\begin{aligned}(2 - 3i)(4 + i) &= 2(4) + 2(i) - 3i(4) - 3i(i)\\ &= 8 + 2i - 12i - 3i^2\\ &= 8 - 10i - 3i^2\end{aligned}$$

35. **Model With Mathematics** The formula $E = IZ$ is used to calculate voltage, where E is voltage, I is current, and Z is impedance. If the voltage in a circuit is $35 + 10i$ volts and the impedance is $4 + 4i$ ohms, what is the current (in amps)? Write your answer in the form $a + bi$.

LESSON 2-5 Completing the Square

Quick Review

Completing the square is a method used to rewrite a quadratic equation as a perfect square trinomial equal to a constant. A perfect square trinomial with the coefficient of x^2 equal to 1 has the form $(x - p)^2$ which is equivalent to $x^2 - 2px + p^2$.

Example

Solve the equation $0 = x^2 - 2x + 4$ by completing the square.

$0 = x^2 - 2x + 4$ .... Write the original equation.

$-4 = x^2 - 2x$ .... Subtract 4 from both sides of the equation.

$1 - 4 = x^2 - 2x + 1$ .... Complete the square

$-3 = (x - 1)^2$ .... Write the right side of the equation as a perfect square.

$\pm\sqrt{-3} = x - 1$ .... Take the square root of each side of the equation.

$1 \pm \sqrt{-3} = x$ .... Add 1 to each side of the equation.

The solutions are $x = 1 \pm \sqrt{-3}$.

Practice & Problem Solving

Rewrite the equations in the form $(x - p)^2 = q$.

36. $0 = x^2 - 16x + 36$

37. $0 = 4x^2 - 28x - 42$

Solve the following quadratic equations by completing the square.

38. $x^2 - 24x - 82 = 0$

39. $-3x^2 - 42x = 18$

40. $4x^2 = 16x + 25$

41. $12 + x^2 = 15x$

42. **Reason** The height, in meters, of a punted football with respect to time is modeled using the function $f(x) = -4.9x^2 + 24.5x + 1$, where x is time in seconds. You determine that the roots of the function $f(x) = -4.9x^2 + 24.5x + 1$ are approximately -0.04 and 5.04. When does the ball hit the ground? Explain.

43. **Make Sense and Persevere** A bike manufacturer can predict profits, P, from a new sports bike using the quadratic function $P(x) = -100x^2 + 46{,}000x - 2{,}100{,}000$, where x is the price of the bike. At what prices will the company make \$0 in profit?

LESSON 2-6 The Quadratic Formula

Quick Review

The **Quadratic Formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provides the solutions of the quadratic equation $ax^2 + bx + c = 0$ for $a \neq 0$. You can calculate the **discriminant** of a quadratic equation to determine the number of real roots.

$b^2 - 4ac > 0$: $ax^2 + bx + c = 0$ has 2 real roots.

$b^2 - 4ac = 0$: $ax^2 + bx + c = 0$ has 1 real root.

$b^2 - 4ac < 0$: $ax^2 + bx + c = 0$ has 2 non-real roots.

Example

How many real roots does $3x^2 - 8x + 1 = 0$ have?

Find the discriminant.

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4(3)(1) \\ &= 64 - 12 \\ &= 52 \end{aligned}$$

Since $52 > 0$, the equation has two real roots.

Practice & Problem Solving

Use the Quadratic Formula to solve the equation.

44. $x^2 - 16x + 24 = 0$ **45.** $x^2 + 5x + 2 = 0$

46. $2x^2 - 18x + 5 = 0$ **47.** $3x^2 - 5x - 19 = 0$

Use the discriminant to identify the number and type of solutions for each equation.

48. $x^2 - 24x + 19 = 0$ **49.** $3x^2 - 8x + 12 = 0$

50. Find the value(s) of k that will cause the equation $4x^2 - kx + 4 = 0$ to have one real solution.

51. Construct Arguments Why does the graph of the quadratic function $f(x) = x^2 + 4x + 5$ cross the y-axis but not the x-axis?

52. Model With Mathematics The function $C(x) = 0.0045x^2 - 0.47x + 139$ models the cost per hour of running a bus between two cities, where x is the speed in kilometers per hour. At what speeds will the cost of running the bus exceed \$130?

LESSON 2-7 Linear-Quadratic Systems

Quick Review

Solutions to a system of equations are points that produces a true statement for all the equations of the system. The solutions on a graph are the coordinates of the intersection points.

Example

Use substitution to solve the system of equations.

$$\begin{cases} y = 2x^2 - 5x + 4 \\ 5x - y = 4 \end{cases}$$

Substitute $2x^2 - 5x + 4$ for y in the second equation.

$$\begin{aligned} 5x - (2x^2 - 5x + 4) &= 4 \\ -2x^2 + 10x - 8 &= 0 \end{aligned}$$

Factor: $-2(x - 1)(x - 4) = 0$

So $x = 1$ and $x = 4$ are solutions.

When $x = 1$, $y = 2(1)^2 - 5(1) + 4 = 1$.

When $x = 4$, $y = 2(4)^2 - 5(4) + 4 = 16$.

The solutions of the system are (1, 1) and (4, 16).

Practice & Problem Solving

Determine the number of solutions of each system of equations.

53. $\begin{cases} y = x^2 - 5x + 9 \\ y = 3 \end{cases}$ **54.** $\begin{cases} y = 3x^2 + 4x + 5 \\ y - 4 = 2x \end{cases}$

Solve each system of equations.

55. $\begin{cases} y = x^2 + 4x + 3 \\ y - 2x = 6 \end{cases}$ **56.** $\begin{cases} y = x^2 + 2x + 7 \\ y = 7 + x \end{cases}$

57. Model With Mathematics An archer shoots an arrow to a height (meters) given by the equation $y = -5t^2 + 18t - 0.25$, where t is the time in seconds. A target sits on a hill represented by the equation $y = 0.75x - 1$. At what height will the arrow strike the target, and how long will it take?