

## Topic Review



## TOPIC ESSENTIAL QUESTION

- How are rational exponents and radical equations used to solve real-world problems?

## Vocabulary Review

Choose the correct term to complete each sentence.

- In the expression  $\sqrt[n]{c}$ ,  $n$  is the \_\_\_\_\_.
- In the expression  $\sqrt[n]{c}$ ,  $c$  is the \_\_\_\_\_.
- Radicals with the same index and the same radicand are \_\_\_\_\_.
- $A(n)$  \_\_\_\_\_ is a function defined by a radical expression.
- $A(n)$  \_\_\_\_\_ of a number  $c$  is  $x$ , such that  $x^n = c$ .
- $A(n)$  \_\_\_\_\_ is a potential solution that must be rejected because it does not satisfy the original equation.
- When all  $n$ th roots of perfect  $n$ th powers have been simplified and no radicals remain in the denominator, an expression is in \_\_\_\_\_.
- $A(n)$  \_\_\_\_\_ results from the application of one function to the output of another function.

- composite function
- extraneous solution
- index
- inverse function
- like radicals
- $n$ th root
- radical function
- radicand
- reduced radical form

## Concepts &amp; Skills Review

## LESSON 5-1

 $n$ th Roots, Radicals, and Rational Exponents

## Quick Review

An  $n$ th root of a number  $c$  is  $x$ , such that  $x^n = c$ . The  $n$ th root of  $c$  can be represented as  $\sqrt[n]{c}$ , where  $n$  is the **index** and  $c$  is the **radicand**.

## Example

Solve the equation  $2x^4 = 162$ .

$$2x^4 = 162 \text{ ..... Write the original equation.}$$

$$x^4 = 81 \text{ ..... Divide both sides by 2.}$$

$$(x^4)^{\frac{1}{4}} = (81)^{\frac{1}{4}} \text{ ..... Raise both sides to the reciprocal of the exponent of } x.$$

$$x = \pm 3 \text{ ..... Use the Power of a Power Property.}$$

## Practice &amp; Problem Solving

What is the value of each expression? Round to the nearest hundredth, if necessary.

10.  $\sqrt[4]{16^2}$

11.  $-\sqrt[3]{25^6}$

Simplify each expression.

12.  $\sqrt[3]{27x^{12}}$

13.  $\sqrt[4]{16a^{24}b^8}$

Solve each equation.

14.  $750 = 6y^3$

15.  $1,280 = 5z^4$

16. **Communicate Precisely** Describe the relationship between a rational exponent and a root of a number  $x$ .

17. **Make Sense and Persevere** The function  $d(t) = 9.8t^2$  represents how far an object falls, in meters, in  $t$  seconds. How long would it take a rock to fall from a height of 300 m? Round to the nearest hundredth of a second.

**Quick Review**

To simplify radical expressions, look for factors that are perfect  $n$ th power factors.

The Product Property of Radicals and Quotient Property of Radicals can also be used to rewrite radical expressions.

**Product Property of Radicals**  $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

**Quotient Property of Radicals**  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

**Example**

What is  $\sqrt[4]{64} \cdot \sqrt[4]{2}$  in reduced radical form?

$\sqrt[4]{64} \cdot \sqrt[4]{2}$  ..... Write the original expression.

$\sqrt[4]{64} \cdot 2$  ..... Use the Product Property of Radicals.

$\sqrt[4]{128}$  ..... Multiply.

$\sqrt[4]{16} \cdot \sqrt[4]{8}$  ..... Rewrite using the Product Property of Radicals.

$2\sqrt[4]{8}$  ..... Simplify.

**Practice & Problem Solving**

What is the reduced radical form of each expression?

18.  $\sqrt{x^6y^4} \cdot \sqrt{x^8y^6}$       19.  $\sqrt[3]{\frac{243m^4}{3m}}$

20.  $\sqrt[3]{5x^4} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{25x^3}$       21.  $\sqrt{\frac{98a^{10}}{2a^4}}$

**Multiply.**

22.  $(\sqrt{n} - \sqrt{7})(\sqrt{n} + 3\sqrt{7})$       23.  $(9x + \sqrt{2})(9x + \sqrt{2})$

24.  $(5\sqrt{3} + 6)(5\sqrt{3} - 6)$       25.  $\sqrt[3]{4}(6\sqrt[3]{2} - 1)$

How can you rewrite each expression so there are no radicals in the denominator?

26.  $\frac{6}{1 + \sqrt{2}}$       27.  $\frac{5}{2 - \sqrt{5}}$

28.  $\frac{4 + \sqrt{6}}{3 - 3\sqrt{6}}$       29.  $\frac{-9x}{\sqrt{x}}$

30. **Error Analysis** Describe and correct the error made in rewriting the radical expression.

$5\sqrt{18} - \sqrt{27} = 7\sqrt{2}$

31. **Reason** A rectangular wall is  $\sqrt{240}$  ft by  $\sqrt{50}$  ft. You need to paint the wall twice to cover the area with two coats of paint. If each can of paint can cover 60 square feet, how many cans of paint will you need?

## LESSON 5-3

## Graphing Radical Functions

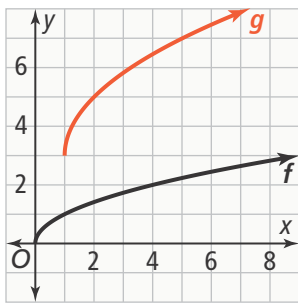
### Quick Review

A **radical function** is a function defined by a radical expression. To determine transformations of a radical function, write the radical function in the form  $h(x) = \sqrt[n]{x-h} + k$  and compare it to the parent function.

### Example

Graph  $g(x) = 2\sqrt{x-1} + 3$ .

$g(x) = 2\sqrt{x-1} + 3$  is a vertical stretch by a factor of 2, a horizontal shift 1 unit to the right, and a vertical shift 3 units up from the parent function  $f(x) = \sqrt{x}$ .



### Practice & Problem Solving

Graph the following functions. What are the domain and range? Is the function increasing or decreasing?

32.  $f(x) = \sqrt{x} - 1$

33.  $f(x) = \sqrt[3]{x} + 2$

34.  $f(x) = \frac{1}{2}\sqrt{x+1}$

35.  $f(x) = 2\sqrt[3]{x} - 1$

36.  $f(x) = \sqrt[3]{x-3}$

37.  $f(x) = \sqrt{x+4} - 2$

38. **Communicate Precisely** Explain how to rewrite the function  $g(x) = \sqrt[3]{8x-24} + 1$  to identify the transformations from the parent graph  $f(x) = \sqrt[3]{x}$ .

39. **Reason** The speed  $s$ , in miles per hour, of a car when it starts to skid can be estimated using the formula  $s = \sqrt{30 \cdot 0.5d}$ , where  $d$  is the length of the skid marks, in feet. Graph the function. If a car's skid marks measure 40 ft in a zone where the speed limit is 25 mph, was the car speeding? Explain.

## LESSON 5-4

## Solving Radical Equations

### Quick Review

To solve a radical equation, isolate the radical. Raise both sides of the equation to the appropriate power to eliminate the radical and solve for  $x$ . Then check for **extraneous solutions**. If the equation includes more than one radical, eliminate one radical at a time using a similar process.

### Example

Solve the radical equation  $\sqrt{6-x} = x$ .

$\sqrt{6-x} = x$  ..... Write the original equation.

$(\sqrt{6-x})^2 = (x)^2$  ..... Square both sides.

$6-x = x^2$  ..... Simplify.

$0 = x^2 + x - 6$  ..... Write in standard form.

$0 = (x+3)(x-2)$  ..... Factor.

$x = -3$  or  $x = 2$  ..... Use the Zero-Product Property.

Check the solutions to see if they both make the original equation true.

### Practice & Problem Solving

Solve each radical equation. Check for extraneous solutions.

40.  $\sqrt[3]{x} - 2 = 7$

41.  $\sqrt{2x} = 12$

42.  $\sqrt{25+x} + 5 = 9$

43.  $13 - \sqrt[4]{x} = 10$

44.  $\sqrt{5x+1} + 1 = x$

45.  $\sqrt{6x-20} - x = -6$

46. **Construct Arguments** Give an example of a radical equation that has no real solutions. Explain your reasoning.

47. **Make Sense and Persevere** The formula  $d = \frac{\sqrt{15w}}{3.14}$  gives the diameter  $d$ , in inches, of a rope needed to lift a weight of  $w$ , in tons. How much weight can be lifted with a rope that has a diameter of 4 in?

## LESSON 5-5

## Function Operations

### Quick Review

You can add, subtract, multiply, or divide functions. When adding and subtracting functions, the domain is the intersection of the domains of the two functions. When multiplying and dividing functions, the domain is the set of all real numbers for which both original functions and the new function are defined. You can also compose functions, by using one function as the input for another function. These are called **composite functions**.

### Example

Let  $f(x) = 5x$  and  $g(x) = 3x - 1$ . What is the rule for the composition  $f \circ g$ ?

$$\begin{aligned} f \circ g &= f(g(x)) \cdots \cdots \text{Apply the definition.} \\ &= f(3x - 1) \cdots \cdots \text{Apply the rule for } g. \\ &= 5(3x - 1) \cdots \cdots \text{Apply the rule for } f. \\ &= 15x - 5 \cdots \cdots \text{Distribute.} \end{aligned}$$

### Practice & Problem Solving

Let  $f(x) = -x + 6$  and  $g(x) = 5x$ . Identify the rule for the following functions.

48.  $f + g$

49.  $f - g$

50.  $g(f(2))$

51.  $f(g(-1))$

52. **Reason** For the functions  $f$  and  $g$ , what is the domain of  $f \circ g$ ?  $\frac{f}{g}$ ?  $\frac{g}{f}$ ?

53. **Make Sense and Persevere** A test has a bonus problem. If you get the bonus problem correct, you will receive 2 bonus points and your test score will increase by 3% of your score. Let  $f(x) = x + 2$  and  $g(x) = 1.03x$ , where  $x$  is the test score without the bonus problem. Find  $g(f(78))$ . What does  $g(f(78))$  represent?

## LESSON 5-6

## Inverse Relations and Functions

### Quick Review

An **inverse relation** is formed when the roles of the independent and dependent variables are reversed. If an inverse relation of a function,  $f$ , is itself a function, it is called the **inverse function** of  $f$ , which is written  $f^{-1}(x)$ .

### Example

What is the inverse of the relation represented in the table?

$x$	$y$
-2	0
-1	6
0	5
1	3
3	-1

Switch the values of  $x$  and  $y$ . Then reorder the ordered pairs.

$x$	$y$
-1	3
0	-2
3	1
5	0
6	-1

### Practice & Problem Solving

Find an equation of the inverse function.

54.  $f(x) = -4x^2 + 3$

55.  $f(x) = \sqrt{x - 4}$

56.  $f(x) = 9x + 5$

57.  $f(x) = \sqrt{x + 7} - 1$

58. **Error Analysis** Jamie said the inverse of  $f(x) = \sqrt{x - 9}$  is  $f^{-1}(x) = (x + 9)^2$ . Is Jamie correct? Explain.

59. **Make Sense and Persevere** An electrician charges \$50 for a house visit plus \$40 per hour. Write a function for the cost  $C$  of an electrician charging for  $h$  hours. Find the inverse of the function. If the bill is \$150, how long did the electrician work?