Topic Review

> **? TOPIC ESSENTIAL QUESTION**
>
> 1. How are trigonometric functions used to solve real-world problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The _____ of an angle in standard position is along the positive x-axis.

3. The _____ of an angle is the other side of an angle in standard position.

4. The distance between the midline and the minimum or maximum of a periodic function is called the _____.

5. The _____ of a periodic function is the reciprocal of the period.

6. A horizontal translation of a periodic function is often called a _____.

7. If an angle θ is in standard position, the _____ for θ is the acute angle formed by the x-axis and the terminal side of θ.

- amplitude
- frequency
- initial side
- phase shift
- reference angle
- terminal side

Concepts & Skills Review

LESSON 7-1 ▸ **Trigonometric Functions and Acute Angles**

Quick Review

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

Example

Write the six trigonometric ratios for the angle θ in the given triangle.

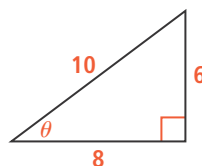$$\sin \theta = \frac{24}{25} \qquad \csc \theta = \frac{25}{24}$$

$$\cos \theta = \frac{7}{25} \qquad \sec \theta = \frac{25}{7}$$

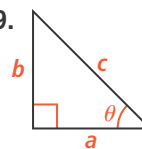$$\tan \theta = \frac{24}{7} \qquad \cot \theta = \frac{7}{24}$$

Practice & Problem Solving

Write the six trigonometric ratios for the angle θ in each given triangle.

8.

9.

What are the trigonometric ratios of θ in a right triangle with the given value?

10. $\sin \theta = \frac{5}{13}$

11. $\cot \theta = \frac{56}{33}$

12. **Look for Relationships** What trigonometric ratio is given by the cofunction identity $\sec (90° - \theta)$?

13. **Make Sense and Persevere** A 15-foot ladder is leaning against the side of a house at a 65° angle. What is the distance from the house to the base of the ladder? Round to the nearest hundredth.

Quick Review

An angle is in **standard position** when its vertex is at the origin and the initial side lies on the *x*-axis. Angles in standard position may be named with positive values or negative values.

The **unit circle** is a circle that has its center at the origin and has a radius of 1. An angle of full circle rotation, or 360°, has a measure of 2π radians.

Example

What is the measure of this angle as a positive number of degrees and in radians? As a negative number of degrees and in radians?
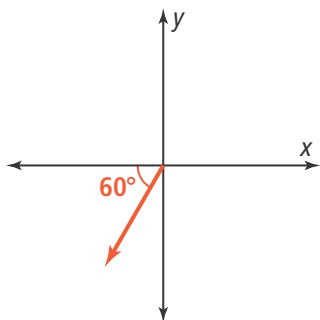
$m\angle\theta = 180° + 60° = 240°$

$= 240°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{3}$

$m\angle\theta = 240° - 360° = -120°$

$= 120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$

Practice & Problem Solving

Find a positive angle measure for each reference angle.

14. 67° in Quadrant I   **15.** 63° in Quadrant IV

16. 25° in Quadrant II   **17.** 14° in Quadrant III

Convert the angle measures.

18. 136° to radians   **19.** $\frac{2\pi}{3}$ radians to degrees

20. 80° to radians   **21.** $-\frac{\pi}{3}$ radians to degrees

For each angle give the reference angle and Quadrant.

22. $-\frac{3\pi}{4}$ radians   **23.** 330°

24. Communicate Precisely Why is it convenient to express an angle in radians when you want to compute arc length?

25. Model With Mathematics The radius of a pond is about 840 feet. After walking around the pond through an angle of $\frac{2\pi}{3}$, you pick up a plastic bottle. You carry it to a recycle bin at a point where you have walked through an angle of $\frac{5\pi}{4}$. How far did you carry the bottle?

LESSON 7-3 · Trigonometric Functions and Real Numbers

Quick Review

The domains of the sine and cosine functions are extended to all real numbers using the unit circle. The coordinates of the point where the terminal side of an angle in standard position intersects the unit circle are $(\cos \theta, \sin \theta)$. The values of the other trigonometric functions can be calculated from this result.

Example

Use the unit circle to evaluate $\tan \frac{\pi}{6}$.
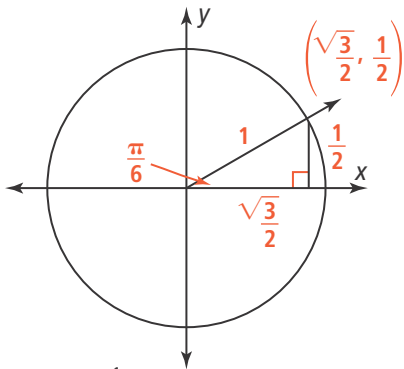
$$\tan \frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Practice & Problem Solving

Find the sine and cosine for each angle.

26. $\frac{4\pi}{3}$

27. $135°$

28. $\frac{5\pi}{6}$

29. $420°$

Find the tangent for each angle.

30. $120°$

31. $-\frac{\pi}{4}$

Find the secant, cosecant, and cotangent for each angle.

32. $-135°$

33. $\frac{8\pi}{3}$

34. **Use Structure** What is $\sin \theta$ if $\cos \theta = \frac{3}{5}$ and θ is in Quadrant IV?

35. **Reason** A scout team is searching a circular region in a 6-mile radius around a camp. Two of the scouts travel on a route that is 45° east of south from the camp. What is their final position, relative to the camp?
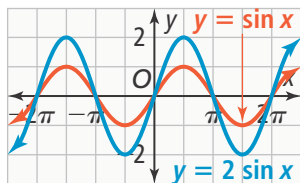

LESSON 7-4 · Graphing Sine and Cosine Functions

Quick Review

The distance between the midline and the minimum or maximum point is the **amplitude**. The **period** is the interval of the domain for which the function does not repeat. **Frequency** is the reciprocal of the period.

Example

What are the amplitude, period, and frequency of $y = 2 \sin x$?

The distance between the midline and maximum point is 2, so the amplitude is 2. The period is 2π. The frequency is 1.

Practice & Problem Solving

What are the amplitude, period, and frequency of each function?
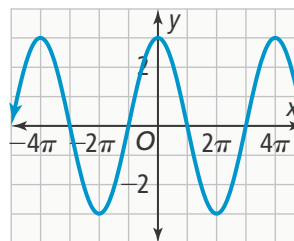
36. $y = \frac{1}{4} \cos (4x)$

37. $y = 3 \sin \left(\frac{1}{2}\right)x$

38. $y = 4 \sin 2x$

39. $y = -2 \cos 6x$

40. **Use Structure** What equation represents the graph?

Graphing Other Trigonometric Functions

Quick Review

When graphing $y = a \tan bx$, a stretches the graph of the parent function vertically and b compresses the graph of the parent function horizontally. The period of the tangent function can be found using period $= \frac{\pi}{|b|}$.
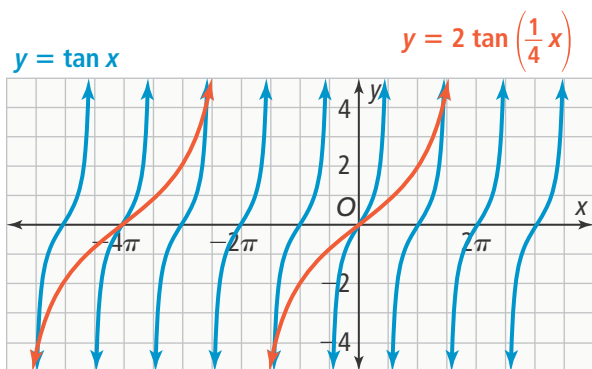
Example

How can you use transformations of the parent function to sketch the graph of the function $y = 2 \tan \frac{1}{4} x$?

$a = 2$, so stretch the graph vertically by a factor of 2.

$b = \frac{1}{4}$, so stretch the graph horizontally by a factor of 4.

$y = \tan x$

$y = 2 \tan \left(\frac{1}{4} x \right)$

Practice & Problem Solving

Sketch the graph of the function. Then describe how the parent graph of the function was affected by the transformations.

41. $y = \frac{1}{4} \tan 2x$ **42.** $y = \frac{1}{2} \cot 3x$

43. Use Structure Describe the domain, range, period, zeros, and asymptotes of the function $y = \cot x$.

44. Reason Write a function that represents the height, h, of the triangle where θ is the angle indicated. Graph the function over the domain $[0, \frac{\pi}{2}]$.

10 ft
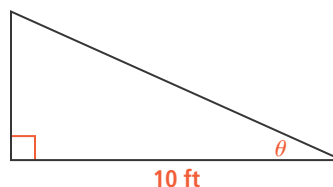
45. Make Sense and Persevere The function $y = 3 \sec \theta$ models the length of a pole leaning against a wall as a function of the measure of the angle θ formed by the pole and the horizontal when the bottom of the pole is 3 ft from the wall. Graph the function and find the length the pole when $\theta = 62°$. Round to the nearest hundredth.

Translating Trigonometric Functions

Quick Review

A horizontal translation of a periodic function is the **phase shift**. When graphing $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, $|a|$ is the amplitude, $\frac{|b|}{2\pi}$ is the frequency, c is the phase shift, and d is the vertical shift.

Example

What are the key features of the function $y = 5 \cos 2(x - 1) + 6$.

$a = 5$, so the amplitude is 5.
$b = 2$, so the frequency is 2, which means the period is $\frac{2\pi}{2} = \pi$.
$c = 1$, so the phase shift is 1 unit to the right.
$d = 6$, so the vertical shift is 6 units up.

Practice & Problem Solving

Identify the amplitude, period, phase shift, and vertical shift of the function.

46. $y = -4 \sin (x + 4\pi) - 8$

47. $y = \frac{1}{4} \cos \left[6 \left(x + \frac{\pi}{2} \right) \right] + 2$

48. Use Structure Write an equation that models the function represented by the graph using the cosine function.

$\left(-\frac{\pi}{4}, 3 \right)$

$\left(\frac{3\pi}{4}, -1 \right)$