TOPIC 8

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do trigonometric identities and equations help you solve problems involving real or complex numbers?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A(n) ________ is a trigonometric equation that is true for all values of the variable for which both sides of the equation are defined.
3. The ________ has two axes, like the Cartesian coordinate plane, which are the real axis and the imaginary axis.
4. The horizontal axis, also called the ________, is for the real part of a complex number.
5. The vertical axis, also called the ________, is for the imaginary part of a complex number.
6. The ________ is the length of the segment from the point that corresponds to the complex number to the origin.
7. The angle θ measured counterclockwise from the positive real axis to the segment is the ________.

- Law of Sines
- Law of Cosines
- trigonometric identity
- complex plane
- real axis
- imaginary axis
- modulus of a complex number
- argument
- polar form of a complex number

Concepts & Skills Review

LESSON 8-1 Solving Trigonometric Equations Using Inverses

Quick Review

An inverse trigonometric function allows you to input the values in a limited range of a trigonometric function and find the corresponding measure of an angle in the domain of the trigonometric function.

Example

Solve the trigonometric equation $5 \sin \theta = 3 \sin \theta + 1$ for values between 0 and 2π.

$5 \sin \theta = 3 \sin \theta + 1$ Write the original equation.

$2 \sin \theta = 1$ Subtract $3 \sin \theta$.

$\sin \theta = \frac{1}{2}$ Divide by 2.

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ Find the sine inverse.

$\theta = \frac{\pi}{6}$ Solve.

Reflect the angle $\frac{\pi}{6}$ across the y-axis; that angle will also have a sine of $\frac{1}{2}$. That angle is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Practice & Problem Solving

Evaluate each function. Angle values must be within the range of each inverse function. Give answers in radians and in degrees.

8. $\tan^{-1}(\sqrt{3})$
9. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$
10. $\tan^{-1}(-1)$
11. $\cos^{-1}\left(-\frac{1}{2}\right)$

Solve each trigonometric equation for values between 0 and 2π.

12. $3 \tan x - \sqrt{3} = 0$
13. $2 \cos x + \sqrt{2} = 0$
14. **Reason** Why is the domain of the inverse sine function restricted to the interval $[-1, 1]$?
15. **Make Sense and Persevere** A bird flies 78 ft from the top of a 4 ft tall bird feeder to the top of a 65 ft tree. To the nearest degree, find the angle of elevation of the line of sight from the top of the bird feeder to the top of the tree.

TOPIC 8 REVIEW

LESSON 8-2 Law of Sines and Law of Cosines

Quick Review

The Law of Sines and the Law of Cosines allow you to apply trigonometric functions to non-right triangles.

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $a^2 = b^2 + c^2 - 2bc(\cos A)$

Example

In $\triangle ABC$, $m\angle A = 93°$, $a = 15$, and $b = 11$. To the nearest degree, what is $m\angle B$?

$\frac{\sin A}{a} = \frac{\sin B}{b}$ ······ Use the Law of Sines.

$\frac{\sin 93}{15} = \frac{\sin B}{11}$ ······ Substitute.

$\frac{11 \sin 93}{15} = \sin B$ ······ Isolate the sine function.

$\sin^{-1}\left(\frac{11 \sin 93}{15}\right) = B$ ······ Use the inverse sine function.

$m\angle B \approx 47°$ ······ Solve.

Practice & Problem Solving

Use the Law of Sines to solve.

16. In $\triangle MNP$, $m\angle N = 112°$, $n = 14$, and $p = 6$. What is $m\angle P$?

17. In $\triangle XYZ$, $m\angle X = 40°$, $m\angle Y = 25°$, and $x = 13$. What is y?

In $\triangle QRS$, find $m\angle Q$.

18. $q = 7$, $r = 6$, $s = 10$

19. $q = 8$, $r = 5$, $s = 6$

20. **Look for Relationships** How do you know whether to use the Law of Sines or the Law of Cosines to solve a problem?

21. **Make Sense and Persevere** Mark went to the beach, parked his car, and walked 500 yd down a path toward the beach. Mark then turned onto a boardwalk at a 125° angle to his path and walked another 140 yd along the boardwalk to a pier. If Mark turns to face his car, what angle does he turn?

LESSON 8-3 Trigonometric Identities

Quick Review

Quotient Identity: $\tan x = \frac{\sin x}{\cos x}$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

Cofunction Identities: $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

Odd-Even Identities: $\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

Sum and Difference Formulas:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Example

What is the simplified form of $\frac{\csc^2 x - 1}{\csc^2 x}$?

$\frac{\csc^2 x - 1}{\csc^2 x} = \frac{\csc^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$ ······ Rewrite the fraction.

$= 1 - \sin^2 x$ ······ Use the definition of sine.

$= \cos^2 x$ ······ Apply the Pythagorean Identity.

Practice & Problem Solving

Use a trigonometric identity to write a different form of each expression.

22. $\tan^2 x + 1$

23. $\tan x + \cot x$

24. $\frac{1 + \tan^2 x}{1 - \tan^2 x}$

25. $\frac{\sec x - 1}{\sec x + 1}$

Find the exact value of each expression. Then evaluate the function on your calculator. Compare the calculator value to your exact value.

26. $\sin 15°$

27. $\cos 105°$

28. $\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

29. $\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

30. **Use Structure** Find expressions for $\sin 2\theta$ and $\cos 2\theta$.

31. **Model With Mathematics** Is a noise with a sound wave modeled by $y = \sin(1{,}500\pi x)$ cancelled out by another noise with a sound wave modeled by $y = \sin\left[1{,}500\pi\left(x - \frac{1}{250}\right)\right]$? Explain.

LESSON 8-4 The Complex Plane

Quick Review

A complex number has the form $a + bi$, where a and b are real numbers. The **complex plane** has two axes. The horizontal axis, or **real axis,** is for the real part of a complex number. The vertical axis, or **imaginary axis,** is for the imaginary part of a complex number. The **modulus of a complex number** is the distance from the point representing the complex number to the origin. The *distance* between two complex numbers is the modulus of the *difference* between the two numbers.

Example

What is the modulus of 4 – 3*i*?

$$|z| = \sqrt{z \bullet \bar{z}}$$
$$= \sqrt{(4 + 3i)(4 - 3i)}$$
$$= \sqrt{16 - 9i^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

Practice & Problem Solving

Find the midpoint of the segment that joins the complex numbers.

32. $-7 + 5i, 3 - 15i$

33. $1 + 9i, 11 - i$

Find the modulus of each complex number.

34. $13 + 8i$

35. $-3 + i$

Find the distance between the complex numbers.

36. $r = -14 + 4i$, $s = -8 + i$

37. $r = 7 + 16i$, $s = -3 - 13i$

38. **Look for Relationships** What is the relationship between the modulus of a complex number and the modulus of its complex conjugate? Explain.

39. **Make Sense and Persevere** On a coordinate plane, a library is located at the coordinates $(-5, 9i)$. The fire station is located at $(7, -7i)$. The school is halfway between the library and the fire station. What are the coordinates of the school?

LESSON 8-5 Polar Form of Complex Numbers

Quick Review

Rectangular form of a complex number: $z = a + bi$
Polar form of a complex number: $z = r \text{ cis } \theta$

$$r = |z| = \sqrt{a^2 + b^2}$$

$a = r\cos\theta$ $\quad$ $b = r\sin\theta$

Convert to polar form: $\tan\theta = \frac{b}{a}$ so $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Convert to rectangular form: $a = r\cos\theta$, $b = r\sin\theta$

Product Formula: $(r \text{ cis } \alpha)(s \text{ cis } \beta) = rs \text{ cis } (\alpha + \beta)$

Power Formula: $z^n = r^n \text{ cis } n\theta$

Example

Express $z = 4 \text{ cis } \frac{\pi}{3}$ in rectangular form.

$a = r\cos\theta$ $\quad$ $b = r\sin\theta$

$a = 4\cos\frac{\pi}{3}$ $\quad$ $b = 4\sin\frac{\pi}{3}$

$a = 2$ $\quad$ $b = 2\sqrt{3}$

$z = a + bi = 2 + 2i\sqrt{3}$

Practice & Problem Solving

Write each expression in rectangular form.

40. $z = 3 \text{ cis } \frac{\pi}{6}$

41. $z = 6 \text{ cis } \frac{2\pi}{3}$

Write each expression in polar form.

42. $z = 3 + 5i$

43. $z = -1 - 2i$

Find the product of z_1z_2.

44. $z_1 = 1 + i\sqrt{2}$ and $z_2 = \sqrt{2} - i$

45. Find z^3 where $z = -2 - 3i$ in rectangular form.

46. **Reason** What complex number can you square to get $4 \text{ cis } \frac{\pi}{2}$? Explain.

47. **Use Structure** Determine the voltage in a circuit when there is a current of $3 \text{ cis } \frac{\pi}{4}$ amps and an impedance of $2 \text{ cis } \frac{2\pi}{3}$ ohms. (*Hint*: Use $E = I \bullet Z$, where E is voltage, I is current, and Z is impedance.)