TOPIC 1

Topic Review

TOPIC ESSENTIAL QUESTION

1. What are different ways in which functions can be used to represent and solve problems involving quantities?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ pairs every input in an interval with the same output value.
3. The point at which a function changes from increasing to decreasing is the __________ of the function.
4. A __________ of a function $y = af(x - h) + k$ is a change made to at least one of the values a, h, and k.
5. A __________ is the value of x when $y = 0$.
6. A __________ is defined by two or more functions, each over a different interval.

- step function
- piecewise-defined function
- minimum
- maximum
- system of linear equations
- transformation
- zero of the function

Concepts & Skills Review

LESSON 1-1 Key Features of Functions

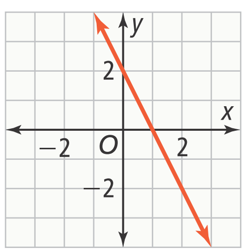
Quick Review

The domain of a function is the set of input values, or x-values. The range of a function is the set of output values, or y-values. These sets can be described using **interval notation** or **set-builder notation.**

A y-intercept is a point on the graph of a function where $x = 0$. An x-intercept is a point on the graph where $y = 0$. An x-intercept may also be a **zero of a function.**

Example

Find the zeros of the function. Then determine over what domain the function is positive or negative.

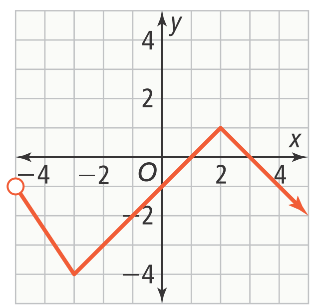


The point where the line crosses the x-axis is (1, 0), so $x = 1$ is a zero of the function. The function is positive on the interval $(-\infty, 1)$ and negative on the interval $(1, \infty)$.

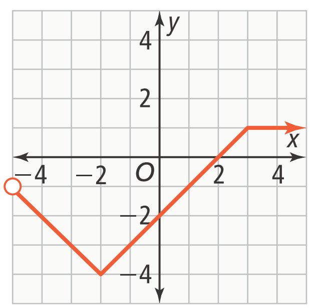
Practice & Problem Solving

Identify the domain and range of the function in set-builder notation. Find the zeros of the function. Then determine for which values of x the function is positive and for which it is negative.

7.



8.



9. **Use Structure** Sketch a graph given the following key features.

   domain: (–5, 5); decreasing: (–3, 1); x-intercepts: –4, –2; positive: (–4, –2)

10. **Communicate Precisely** Jeffrey is emptying a 50 ft^3 container filled with water at a rate of 0.5 ft^3/min. Find and interpret the key features for this situation.

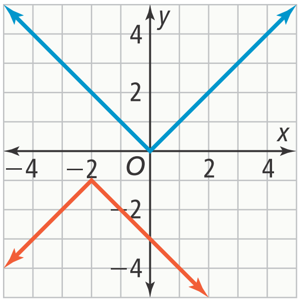
TOPIC 1 REVIEW

Quick Review

There are different types of **transformations** that change the graph of the parent function. A **translation** shifts each point on a graph the same distance and direction. A **reflection** maps each point to a new point across a given line. A **stretch** or a **compression** increases or decreases the distance between the points of a graph and a given line by the same factor.

Example

Graph the parent function $f(x) = |x|$ and $g(x) = -|x + 2| - 1$. Describe the transformation.



Multiplying the absolute value expression by −1 indicates a reflection over the x-axis.

Adding 2 to x indicates a translation 2 units to the left and subtracting 1 from the absolute value expression indicates a translation 1 unit down.

So the graph of g is a reflection of the graph of the parent function f over the x-axis, and then a translation 2 units left and 1 unit down.

Practice & Problem Solving

Graph each function as a translation of its parent function, *f*.

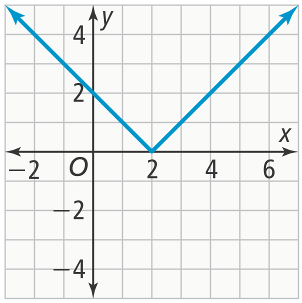
11. $g(x) = |x| - 7$

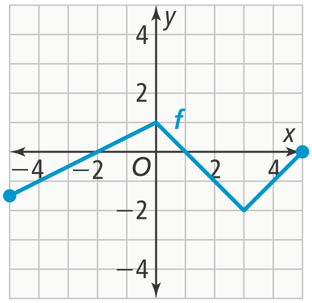
12. $g(x) = x^2 + 5$

Graph the function, *g*, as a reflection of the graph of *f* across the given axis.

13. across the x-axis

14. across the y-axis





15. Look for Relationships Describe the effect of a vertical stretch by a factor greater than 1 on the graph of the absolute value function. How is that different from the effect of a horizontal stretch by the same factor?

16. Use Structure Graph the function that is a vertical stretch by a factor of 3.5 of the parent function $f(x) = |x|$.

17. Use Structure Graph the function that is a horizontal translation 1 unit to the right of the parent function $f(x) = x^2$.

Quick Review

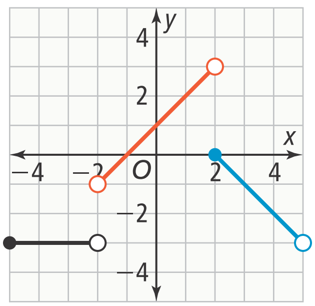
A **piecewise-defined function** is a function defined by two or more function rules over different intervals. A **step function** pairs every number in an interval with a single value. The graph of a step function can look like the steps of a staircase.

Example

Graph the function.

$$y = \begin{cases} -3, \text{ if } -5 \leq x < -2 \\ x + 1, \text{ if } -2 < x < 2 \\ -x + 2, \text{ if } 2 \leq x < 5 \end{cases}$$

State the domain and range. Identify whether the function is increasing, constant, or decreasing on each interval of the domain.



Graph the function.

Domain: $-5 \leq x < -2$ and $-2 < x < 5$

Range: $-3 \leq y < 3$

Increasing when $-2 < x < 3$

Constant when $-5 \leq x < -2$

Decreasing when $2 \leq x < 5$

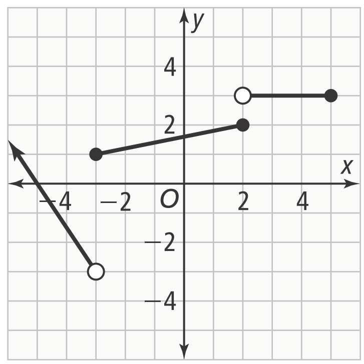
Practice & Problem Solving

Graph each function.

18. $y = \begin{cases} -3, \text{ if } -4 \leq x < -2 \\ -1, \text{ if } -2 \leq x < 0 \\ 1, \text{ if } 0 \leq x < 2 \\ 3, \text{ if } 2 \leq x < 4 \end{cases}$

19. $y = \begin{cases} 2x + 5, \text{ if } x < -3 \\ -x - 2, \text{ if } -3 \leq x < 1 \\ x - 3, \text{ if } x \geq 1 \end{cases}$

20. What rule defines the function in the following graph?



21. Generalize Can every transformation of the absolute value function also be written as a piecewise-defined function? Explain.

22. Model With Mathematics A coach is trying to decide how many new uniforms to purchase for a softball team. If the coach orders more than 10 uniforms, the cost for the extra uniforms is 0.75 times the normal cost per uniform of \$120. Write a piecewise-defined function that gives the cost *C*, in dollars, in terms of the number of uniforms *n* the coach purchases. Determine how much the coach will pay for 18 uniforms.

LESSON 1-4 Arithmetic Sequences and Series

Quick Review

An **arithmetic sequence** is a sequence with a constant difference between consecutive terms. This difference is known as the **common difference.**

recursive definition: $a_n = \begin{cases} a_1, \text{ if } n = 1 \\ a_{n-1} + d, \text{ if } n > 1 \end{cases}$

explicit definition: $a_n = a_1 + (n - 1)d$

A **finite arithmetic series** is the sum of all the numbers in an arithmetic sequence.

Example

Given the sequence 22, 17, 12, 7, ..., write the explicit formula. Then find the 6th term.

$d = -5$ .... Find the common difference.

$a_n = 22 + (n - 1)(-5)$ .... Substitute 22 for a_1 and -5 for d.

$a_n = 22 - 5(n - 1)$ .... Simplify.

$a_6 = 22 - 5(6 - 1)$ .... Substitute 6 for n.

$a_6 = -3$ .... Solve for the 6th term.

Practice & Problem Solving

What is the common difference and the next term in the arithmetic sequence?

23. 3, 15, 27, 39, ...

24. 19, 13, 7, 1, ...

What are the recursive and explicit functions for each sequence?

25. 5, 9, 13, 17, 21, ...

26. 25, 18, 11, 4, −3, ...

Find the sum of an arithmetic sequence with the given number of terms and values of a_1 and a_n.

27. 8 terms, $a_1 = 2$, $a_8 = 74$

28. 12 terms, $a_1 = 87$, $a_{12} = 10$

What is the value of each of the following series?

29. $\sum_{n=1}^{9} (1 + 3n)$

30. $\sum_{n=1}^{6} (5n - 2)$

31. **Make Sense and Persevere** Cubes are stacked in the shape of a pyramid. The top row has 1 cube, the second row has 3, and the third row has 5. If there are 9 rows of cubes, how many cubes were used to make the front of the pyramid?

LESSON 1-5 Solving Equations and Inequalities by Graphing

Quick Review

To solve an equation by graphing, write two new equations by setting y equal to each expression in the original equation. Approximate coordinates of any points of intersection. The x-values of these points are the solutions to the equation. You can also solve equations using tables or graphing technology.

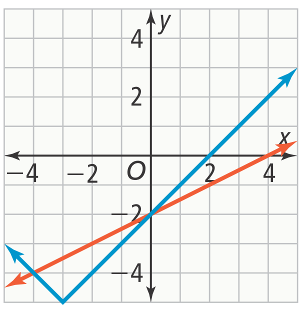
Example

Solve $|x + 3| - 5 = \frac{1}{2}x - 2$ by graphing.

Graph $y = |x + 3| - 5$ and $y = \frac{1}{2}x - 2$.

It appears that $x = -4$ and $x = 0$ are solutions.

Confirm the solutions by substituting into the original equation.



Practice & Problem Solving

Use a graph to solve each equation.

32. $-x + 2 = x^2$

33. $\frac{1}{4}|x + 3| = 2$

Use a graph to solve each inequality.

34. $x^2 + 2x - 3 > 0$

35. $x^2 - 7x - 8 < 0$

36. **Construct Arguments** Is graphing always the most convenient method for solving an equation? Why or why not?

37. **Model With Mathematics** A truck is traveling 30 mi ahead of a car at an average rate of 55 mph. The car is traveling at a rate of 63 mph. Let x represent the number of hours that the car and truck travel. Write an inequality to determine at what times the car will be ahead of the truck and graph the inequality to solve.

LESSON 1-6 Linear Systems

Quick Review

A **system of linear equations** is a set of two or more equations using the same variables. The **solution of a system of linear equations** is the set of all ordered coordinates that simultaneously make all equations in the system true. A **system of linear inequalities** is a set of two or more inequalities using the same variables.

Example

Solve the system. $\begin{cases} -4x + 4y = 16 \\ -x + 2y = 10 \end{cases}$

$x = 2y - 10$ ······ Solve the second equation for x.

$-4(2y - 10) + 4y = 16$ ······ Substitute $2y - 10$ for x. Solve for y.
$y = 6$

$x = 2(6) - 10$ ······ Substitute 6 for y in the equation $x = 2y - 10$.
$x = 2$

Practice & Problem Solving

Solve each system of equations.

38. $\begin{cases} y = 2x + 5 \\ 2x + 4y = 10 \end{cases}$

39. $\begin{cases} y = 2x - 6 \\ 6x + y = 10 \end{cases}$

40. Use Structure Write a linear system in two variables that has infinitely many solutions.

41. Model With Mathematics It takes Leo 12 h to make a table and 20 h to make a chair. In 8 wk, Leo wants to make at least 5 tables and 8 chairs to display in his new shop. Leo works 40 h a week. Write a system of linear inequalities relating the number of tables x and the number of chairs y Leo will be able to make. List two different combinations of tables and chairs Leo could have to display at the opening of his new shop.

LESSON 1-7 Solving Linear Systems Using Matrices

Quick Review

You can solve systems with matrices. A matrix is a rectangular array of numbers, usually shown inside square brackets. Row operations can be applied to a matrix to create an equivalent matrix and can be used to write the matrix in **reduced row echelon form.**

Example

Solve the system $\begin{cases} -2x + 8y = 10 \\ 4x - 3y = 6 \end{cases}$ **using a matrix.**

$\left[\begin{array}{rr|r} -2 & 8 & 10 \\ 4 & -3 & 6 \end{array}\right]$ ······ Write the system in matrix form.

$\left[\begin{array}{rr|r} 1 & -4 & -5 \\ 4 & -3 & 6 \end{array}\right]$ ······ Divide row_1 by -2.

$\left[\begin{array}{rr|r} 1 & -4 & -5 \\ 0 & 13 & 26 \end{array}\right]$ ······ Multiply row_1 by -4, and add to row_2.

$\left[\begin{array}{rr|r} 1 & -4 & -5 \\ 0 & 1 & 2 \end{array}\right]$ ······ Divide row_2 by 13.

$\left[\begin{array}{rr|r} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$ ······ Multiply row_2 by 4, and add to row_1.

The solution to the system of linear equations is $x = 3$ and $y = 2$.

Practice & Problem Solving

42. Write the matrix that represents the system of equations and find the reduced row echelon form. $\begin{cases} 4x + 8y = 12 \\ -2x - 6y = 32 \end{cases}$

43. Write a system of equations represented by the matrix. $\left[\begin{array}{rr|r} 5 & 2 & 6 \\ 6 & -7 & -4 \end{array}\right]$

44. Communicate Precisely Why is it important to write equations in standard form before entering the coefficients into a matrix?

45. Model With Mathematics A trivia game consists of three types of questions in three different colors: red, white, and blue. Each type of question is worth a different number of points. Holly answered 4 red, 1 white, and 1 blue question correctly and earned 23 points. Jung answered 5 white and 1 blue question correctly and earned 35 points. Rochelle answered 2 red and 3 white questions, and earned 19 points. How many points is each color worth?

TOPIC 2

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use quadratic functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. According to the __________, a product is 0 only if one (or more) of its factors is 0.
3. The __________ of a quadratic function is $y = a(x - h)^2 + k$.
4. The __________ of a quadratic function is the value of the radicand, $b^2 - 4ac$.
5. A number with both real and imaginary parts is called a __________.
6. The __________ of a quadratic function is $y = ax^2 + bx + c$.
7. __________ is a method used to rewrite an equation as a perfect square trinomial equal to a constant.

- completing the square
- complex number
- discriminant
- imaginary number
- parabola
- quadratic function
- standard form
- vertex form
- Zero Product Property

Concepts & Skills Review

LESSON 2-1 Vertex Form of a Quadratic Function

Quick Review

The parent **quadratic function** is $f(x) = x^2$. The graph of the function is represented by a **parabola**. All quadratic functions are transformations of $f(x) = x^2$.

The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex of a parabola.

Example

What is the equation of a parabola with vertex (3, 1) and *y*-intercept 10?

$y = a(x - 3)^2 + 1$ ········ Substitute $(h, k) = (2, 3)$.

$10 = a(0 - 3)^2 + 1$ ········ Substitute *y*-intercept (0, 10).

$9 = a(-3)^2$ ········ Simplify.

$9 = 9a$

$a = 1$ ········ Solve for *a*.

$y = 1(x - 3)^2 + 1$ ········ Substitute *a*.

The equation of the parabola is $y = (x - 3)^2 + 1$.

Practice & Problem Solving

Describe the transformation of the parent function $f(x) = x^2$. Then graph the given function.

8. $g(x) = (x + 2)^2 - 4$
9. $h(x) = -2(x - 1)^2 + 5$

Identify the vertex, axis of symmetry, maximum or minimum, domain, and range of each function.

10. $g(x) = -(x + 3)^2 + 2$
11. $h(x) = 3(x - 4)^2 - 3$

Write the equation of each quadratic function in vertex form.

12. Vertex: (2, 1); Point (0, 4)
13. Vertex: (1, 5); Point (3, −1)
14. **Use Structure** The graph of the function $f(x) = x^2$ will be translated 4 units down and 2 units right. What is the resulting function $g(x)$?
15. **Make Sense and Persevere** Find three additional points on the parabola that has vertex (5, 3) and passes through (2, 21).

LESSON 2-2 Standard Form of a Quadratic Function

Quick Review

The **standard form of a quadratic function** is $y = ax^2 + bx + c$ where a, b, and c are real numbers, and $a \neq 0$. Use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex and the axis of symmetry. Substitue 0 for x to find the y-intercept of the quadratic function.

Example

The function $y = -8x^2 + 880x - 5{,}000$ can be used to predict the profits for a company that sells eBook readers for a certain price, x. What is the maximum profit the company can expect to earn?

The maximum value of a quadratic function occurs at the vertex of a parabola. Use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex.

$h = -\frac{880}{2(-8)}$ ······ Substitute −8 for a and 880 for b.

$h = 55$ ······ Simplify.

$x = 55$ ······ Substitute h for x.

$y = -8(55)^2 + 880(55) - 5{,}000$ ······ Substitute 55 for x.

$y = 19{,}200$ ······ Simplify.

The vertex is (55, 19,200). The selling price of \$55 per item gives the maximum profit of \$19,200.

Practice & Problem Solving

Find the vertex and y-intercept of the quadratic function, and use them to graph the function.

16. $y = x^2 - 6x + 15$

17. $y = 4x^2 - 15x + 9$

Write an equation in standard form for the parabola that passes through the given points.

18. (1, 5), (3, 7), (6, 25)

19. (−2, 64), (3, −16), (7, 28)

20. **Higher Order Thinking** A golfer is on a hill that is 60 meters above the hole. The path of the ball can be modeled by the equation $y = -5x^2 + 40x + 60$, where x is the horizontal and y the vertical distance traveled by the ball in meters. How would you use the function to find the horizontal distance traveled by the ball and its maximum height?

21. **Make Sense and Persevere** The number of issues sold per month of a new magazine (in thousands) and its profit (in thousands of dollars) could be modeled by the function $y = -6x^2 + 36x + 50$. Determine the maximum profit.

LESSON 2-3 Factored Form of a Quadratic Function

Quick Review

Factor a quadratic equation by first setting the quadratic expression equal to 0. Then factor and use the **Zero Product Property** to solve. According to the Zero Product Property, if $ab = 0$, then $a = 0$ or $b = 0$ (or $a = 0$ and $b = 0$).

Example

Solve the equation $x^2 + x = 72$.

$x^2 + x - 72 = 0$ ······ Set equation equal to 0.

$(x + 9)(x - 8)$ ······ Factor.

$x + 9 = 0$ or $x - 8 = 0$ ······ Zero Product Property.

$x = -9$ or $x = 8$ ······ Solve.

The solutions for equation $x^2 + x = 72$ are $x = -9$ or $x = 8$.

Practice & Problem Solving

Solve each quadratic equation.

22. $x^2 - 6x - 27 = 0$

23. $x^2 = 7x - 10$

24. $4x^2 + 4x = 3$

25. $5x^2 - 19x = -12$

Identify the interval(s) on which each function is positive.

26. $y = x^2 - x - 30$

27. $y = x^2 + 11x + 28$

28. **Generalize** For what values of x is the expression $(x + 6)^2 > 0$?

29. **Model With Mathematics** A prairie dog burrow has openings to the surface which, if they were graphed, correspond to points (2.5, 0) and (8, 0). What equation models the burrow if, at its deepest, it passes through point (5, −15)?

LESSON 2-4 Complex Numbers and Operations

Quick Review

The **imaginary unit** i is the number whose square is equal to -1. An **imaginary number** bi is the product of any real number b and the imaginary unit i. A **complex number** is a number that may be written in the form $a + bi$. **Complex conjugates** are complex numbers with equivalent real parts and opposite imaginary parts.

Example

Write the product of $3.5i(4 - 6i)$ in the form $a + bi$.

$3.5i(4 - 6i)$

$= 3.5i(4) + 3.5i(-6i)$ ······ Distribute.

$= 14i - 21i^2$ ······ Simplify.

$= 14i - 21(-1)$ ······ Substitute -1 for i.

$= 14i + 21$ ······ Write in the form $a + bi$.

The product is $14i + 21$.

Practice & Problem Solving

Write each product in the form $a + bi$.

30. $(5 - 3i)(2 + i)$

31. $(-3 + 2i)(2 - 3i)$

Divide. Write the answer in the form $a + bi$.

32. $\frac{5}{3 + i}$

33. $\frac{2 - 3i}{1 + 2i}$

34. Error Analysis Describe and correct the error a student made when multiplying complex numbers.

$(2 - 3i)(4 + i) = 2(4) + 2(i) - 3i(4) - 3i(i)$
$= 8 + 2i - 12i - 3i^2$
$= 8 - 10i - 3i^2$

35. Model With Mathematics The formula $E = IZ$ is used to calculate voltage, where E is voltage, I is current, and Z is impedance. If the voltage in a circuit is $35 + 10i$ volts and the impedance is $4 + 4i$ ohms, what is the current (in amps)? Write your answer in the form $a + bi$.

LESSON 2-5 Completing the Square

Quick Review

Completing the square is a method used to rewrite a quadratic equation as a perfect square trinomial equal to a constant. A perfect square trinomial with the coefficient of x^2 equal to 1 has the form $(x - p)^2$ which is equivalent to $x^2 - 2px + p^2$.

Example

Solve the equation $0 = x^2 - 2x + 4$ by completing the square.

$0 = x^2 - 2x + 4$ ······ Write the original equation.

$-4 = x^2 - 2x$ ······ Subtract 4 from both sides of the equation.

$1 - 4 = x^2 - 2x + 1$ ······ Complete the square

$-3 = (x - 1)^2$ ······ Write the right side of the equation as a perfect square.

$\pm\sqrt{-3} = x - 1$ ······ Take the square root of each side of the equation.

$1 \pm \sqrt{-3} = x$ ······ Add 1 to each side of the equation.

The solutions are $x = 1 \pm \sqrt{-3}$.

Practice & Problem Solving

Rewrite the equations in the form $(x - p)^2 = q$.

36. $0 = x^2 - 16x + 36$

37. $0 = 4x^2 - 28x - 42$

Solve the following quadratic equations by completing the square.

38. $x^2 - 24x - 82 = 0$

39. $-3x^2 - 42x = 18$

40. $4x^2 = 16x + 25$

41. $12 + x^2 = 15x$

42. Reason The height, in meters, of a punted football with respect to time is modeled using the function $f(x) = -4.9x^2 + 24.5x + 1$, where x is time in seconds. You determine that the roots of the function $f(x) = -4.9x^2 + 24.5x + 1$ are approximately -0.04 and 5.04. When does the ball hit the ground? Explain.

43. Make Sense and Persevere A bike manufacturer can predict profits, P, from a new sports bike using the quadratic function $P(x) = -100x^2 + 46{,}000x - 2{,}100{,}000$, where x is the price of the bike. At what prices will the company make \$0 in profit?

LESSON 2-6 The Quadratic Formula

Quick Review

The **Quadratic Formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provides the solutions of the quadratic equation $ax^2 + bx + c = 0$ for $a \neq 0$. You can calculate the **discriminant** of a quadratic equation to determine the number of real roots.

$b^2 - 4ac > 0$: $ax^2 + bx + c = 0$ has 2 real roots.

$b^2 - 4ac = 0$: $ax^2 + bx + c = 0$ has 1 real root.

$b^2 - 4ac < 0$: $ax^2 + bx + c = 0$ has 2 non-real roots.

Example

How many real roots does $3x^2 - 8x + 1 = 0$ have?

Find the discriminant.

$$\begin{aligned} b^2 - 4ac &= (-8)^2 - 4(3)(1) \\ &= 64 - 12 \\ &= 52 \end{aligned}$$

Since $52 > 0$, the equation has two real roots.

Practice & Problem Solving

Use the Quadratic Formula to solve the equation.

44. $x^2 - 16x + 24 = 0$

45. $x^2 + 5x + 2 = 0$

46. $2x^2 - 18x + 5 = 0$

47. $3x^2 - 5x - 19 = 0$

Use the discriminant to identify the number and type of solutions for each equation.

48. $x^2 - 24x + 19 = 0$

49. $3x^2 - 8x + 12 = 0$

50. Find the value(s) of k that will cause the equation $4x^2 - kx + 4 = 0$ to have one real solution.

51. **Construct Arguments** Why does the graph of the quadratic function $f(x) = x^2 + 4x + 5$ cross the y-axis but not the x-axis?

52. **Model With Mathematics** The function $C(x) = 0.0045x^2 - 0.47x + 139$ models the cost per hour of running a bus between two cities, where x is the speed in kilometers per hour. At what speeds will the cost of running the bus exceed \$130?

LESSON 2-7 Linear-Quadratic Systems

Quick Review

Solutions to a system of equations are points that produces a true statement for all the equations of the system. The solutions on a graph are the coordinates of the intersection points.

Example

Use substitution to solve the system of equations.

$$\begin{cases} y = 2x^2 - 5x + 4 \\ 5x - y = 4 \end{cases}$$

Substitute $2x^2 - 5x + 4$ for y in the second equation.

$$\begin{aligned} 5x - (2x^2 - 5x + 4) &= 4 \\ -2x^2 + 10x - 8 &= 0 \end{aligned}$$

Factor: $-2(x - 1)(x - 4) = 0$

So $x = 1$ and $x = 4$ are solutions.

When $x = 1$, $y = 2(1)^2 - 5(1) + 4 = 1$.

When $x = 4$, $y = 2(4)^2 - 5(4) + 4 = 16$.

The solutions of the system are (1, 1) and (4, 16).

Practice & Problem Solving

Determine the number of solutions of each system of equations.

53. $\begin{cases} y = x^2 - 5x + 9 \\ y = 3 \end{cases}$

54. $\begin{cases} y = 3x^2 + 4x + 5 \\ y - 4 = 2x \end{cases}$

Solve each system of equations.

55. $\begin{cases} y = x^2 + 4x + 3 \\ y - 2x = 6 \end{cases}$

56. $\begin{cases} y = x^2 + 2x + 7 \\ y = 7 + x \end{cases}$

57. **Model With Mathematics** An archer shoots an arrow to a height (meters) given by the equation $y = -5t^2 + 18t - 0.25$, where t is the time in seconds. A target sits on a hill represented by the equation $y = 0.75x - 1$. At what height will the arrow strike the target, and how long will it take?

TOPIC 2 REVIEW

TOPIC 3

Topic Review

TOPIC ESSENTIAL QUESTION

1. What can the rule for a polynomial function reveal about its graph, and what can the graphs of polynomial functions reveal about the solutions of polynomial equations?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ is the greatest power of the variable in a polynomial expression.
3. The __________ is the non-zero constant multiplied by the greatest power of the variable in a polynomial expression.
4. The __________ of a function describes what happens to its graph as x approaches positive and negative infinity.
5. __________ is the triangular pattern of numbers where each number is the sum of two numbers above it.
6. The __________ determines whether the graph of the function will cross the x-axis at the point or merely touch it.
7. The __________ is a formula that can be used to expand powers of binomial expressions.
8. __________ is a method to divide a polynomial by a linear factor whose leading coefficient is 1.

- Binomial Theorem
- degree of a polynomial
- end behavior
- even function
- Factor Theorem
- identity
- leading coefficient
- multiplicity of a zero
- Pascal's Triangle
- synthetic division

Concepts & Skills Review

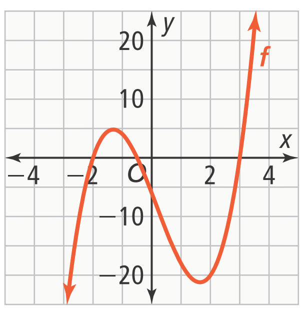
LESSON 3-1 Graphing Polynomial Functions

Quick Review

A **polynomial** can be either a monomial or a sum of monomials. When a polynomial has more than one monomial, the monomials are also referred to as **terms.**

Example

Graph the function $f(x) = 2x^3 - x^2 - 13x - 6$.



There are zeros at $x = -2$, $x = -0.5$, and $x = 3$.

There are turning points between –2 and –0.5 and between –0.5 and 3.

As $x \to -\infty$, $y \to -\infty$.

As $x \to +\infty$, $y \to +\infty$.

Practice & Problem Solving

Graph the polynomial function. Estimate the zeros and the turning points of the graph.

9. $f(x) = x^5 + 2x^4 - 10x^3 - 20x^2 + 9x + 18$

10. $f(x) = x^4 + x^3 - 16x^2 - 4x + 48$

11. **Reason** A polynomial function has the following end behavior: As $x \to -\infty$, $y \to +\infty$. As $x \to +\infty$, $y \to -\infty$. Describe the degree and leading coefficient of the polynomial function.

12. **Make Sense and Persevere** After x hours of hiking, Sadie's elevation is $p(x) = -x^3 + 11x^2 - 34x + 24$, in meters. After how many hours will Sadie's elevation be 18 m below sea level? What do the x- and y-intercepts of the graph mean in this context?

LESSONS 3-2 & 3-3 Adding, Subtracting, and Multiplying Polynomials and Polynomial Identities

Quick Review

To add or subtract polynomials, add or subtract like terms. To multiply polynomials, use the Distributive Property.

Polynomial identities can be used to factor or multiply polynomials.

Example

Add $(-2x^3 + 5x^2 + 2x - 3) + (x^3 - 6x^2 + x + 12)$.

Use the Commutative and Associative Properties. Then combine like terms.

$(-2x^3 + 5x^2 + 2x - 3) + (x^3 - 6x^2 + x - 12)$

$= (-2x^3 + x^3) + (5x^2 - 6x^2) + (2x + x) + (-3 + 12)$

$= -x^3 - x^2 + 3x + 9$

Example

Use polynomial identities to factor $8x^3 + 27y^3$.

Use the Sum of Cubes Identity. Express each term as a square. Then write the factors.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$8x^3 + 27y^3 = (2x)^3 + (3y)^3$

$= (2x + 3y)(4x^2 - 6xy + 9y^2)$

Practice & Problem Solving

Add or subtract the polynomials.

13. $(-8x^3 + 7x^2 + x - 9) + (5x^3 + 3x^2 - 2x - 1)$

14. $(9y^4 - y^3 + 4y^2 + y - 2) - (2y^4 - 3y^3 + 6y - 7)$

Multiply the polynomials.

15. $(9x - 1)(x + 5)(7x + 2)$

Use polynomial identities to multiply each polynomial.

16. $(5x + 8)^2$

17. $(7x - 4)(7x + 4)$

Factor the polynomial.

18. $x^6 - 64$

19. $27x^3 + y^6$

Use Pascal's Triangle or the Binomial Theorem to expand the expressions.

20. $(x - 2)^4$

21. $(x + 5y)^5$

22. **Communicate Precisely** Explain why the set of polynomials is closed under subtraction.

23. **Reason** The length of a rectangle is represented by $3x^3 - 2x^2 + 10x - 4$, and the width is represented by $-x^3 + 6x^2 - x + 8$. What is the perimeter of the rectangle?

LESSON 3-4 Dividing Polynomials

Quick Review

Polynomials can be divided using long division or synthetic division. **Synthetic division** is a method to divide a polynomial by a linear factor whose leading coefficient is 1.

Example

Use synthetic division to divide $x^4 - 5x^3 - 6x^2 + 2x - 8$ by $x + 3$.

−3	1	−5	−6	2	−8
		−3	24	−54	156
	1	−8	18	−52	148
	↓	↓	↓	↓	↓
	x^3	$-8x^2$	$+18x$	-52	$+\frac{148}{x+3}$

The quotient is $x^3 - 8x^2 + 18x - 52$, and the remainder is 148.

Practice & Problem Solving

Use long division to divide.

24. $x^4 + 2x^3 - 8x^2 - 3x + 1$ divided by $x + 2$

Use synthetic division to divide.

25. $x^4 + 5x^3 + 7x^2 - 2x + 17$ divided by $x - 3$

26. **Make Sense and Persevere** A student divided $f(x) = x^3 + 8x^2 - 9x - 3$ by $x - 2$ and got a remainder of 19. Explain how the student could verify the remainder is correct.

27. **Reason** The area of a rectangle is $4x^3 + 14x^2 - 18$ in.². The length of the rectangle is $x + 3$ in. What is the width of the rectangle?

LESSONS 3-5 & 3-6 Zeros of Polynomial Functions and Theorems about Roots of Polynomial Equations

Quick Review

You can factor and use synthetic division to find zeros of polynomial functions. Then you can use the zeros to sketch a graph of the function.

The **Rational Root Theorem** states that the possible rational roots, or zeros, of a polynomial equation with integer coefficients come from the list of numbers of the form: $\pm\frac{\text{factor of } a_0}{\text{factor of } a_n}$.

Example

List all the possible rational solutions for the equation $0 = 2x^3 + x^2 - 7x - 6$. Then find all of the rational roots.

$\pm1, \pm2, \pm3, \pm6$ Factors of the constant term

$\pm1, \pm2$ Factors of the leading coefficient

List the possible roots, eliminating duplicates.

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{6}{1}$

Use synthetic division to find that the roots are $-\frac{3}{2}$, -1, and 2.

Practice & Problem Solving

Sketch the graph of the function.

28. $f(x) = 2x^4 - x^3 - 32x^2 + 31x + 60$

29. $g(x) = x^3 - x^2 - 20x$

30. What x-values are solutions to the equation $x^3 + 2x^2 - 4x + 8 = x^2 - x + 4$?

31. What values of x are solutions to the inequality $x^3 + 3x^2 - 4x - 12 > 0$?

32. What are all of the real and complex roots of the function $f(x) = x^4 - 4x^3 + 4x^2 - 36x - 45$?

33. A polynomial function Q of degree 4 with rational coefficients has zeros $1 + \sqrt{5}$ and $-7i$. What is an equation for Q?

34. **Reason** What does the graph of a function tell you about the multiplicity of a zero?

35. **Make Sense and Persevere** A storage unit in the shape of a rectangular prism measures $2x$ ft long, $x + 8$ ft wide, and $x + 9$ ft tall. What are the dimensions of the storage unit, in feet, if its volume is 792 ft^3?

LESSON 3-7 Transformations of Polynomial Functions

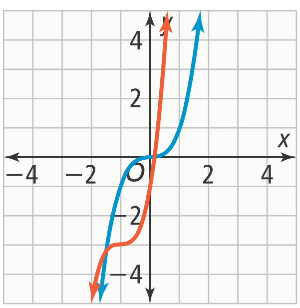
Quick Review

Polynomial functions can be translated, reflected, and stretched in similar ways to other functions you have studied.

Example

How does the graph of $f(x) = 2(x + 1)^3 - 3$ compare to the graph of the parent function?

Parent function: $y = x^3$



Adding 1 shifts the graph to the left 1 unit.

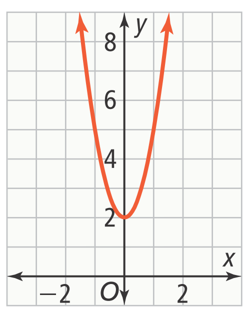
Multiplying by 2 stretches the graph vertically.

Subtracting 3 shifts the graph down 3 units.

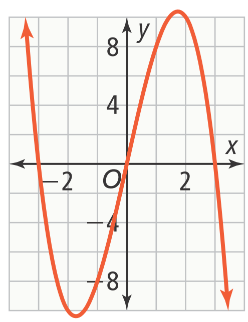
Practice & Problem Solving

Classify each function as even, odd, or neither.

36.



37.



38. **Error Analysis** A student says the graph of $f(x) = 0.5x^4 + 1$ is a vertical stretch and a translation up 1 unit of the parent function. Explain the student's error.

39. **Make Sense and Persevere** The volume of a refrigerator, in cubic centimeters, is given by the function $V(x) = (x)(x + 1)(x - 2)$. Write a new function for the volume of the refrigerator in cubic millimeters if x is in centimeters.

TOPIC 4

Topic Review

? TOPIC ESSENTIAL QUESTION

1. How do you calculate with functions defined as quotients of polynomials, and what are the key features of their graphs?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ can be represented by the equation $y = \frac{1}{x}$.
3. A(n) __________ is any function $R(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.
4. __________ can be modeled by the equation $y = \frac{k}{x}$.
5. A(n) __________ is the quotient of two polynomials.
6. A(n) __________ is a line that a graph approaches but may not touch.
7. A(n) __________ is a fraction that has one or more fractions in the numerator and/or the denominator.
8. A(n) __________ is a value that is a solution to an equation that is derived from an original equation but does not satisfy the original equation.

- inverse variation
- constant of variation
- reciprocal function
- asymptote
- rational function
- extraneous solution
- rational expression
- compound fraction

Concepts & Skills Review

LESSON 4-1 Inverse Variation and the Reciprocal Function

Quick Review

The equation $y = \frac{k}{x}$, or $xy = k$, $k \neq 0$, represents an **inverse variation**, where k is the **constant of variation**. The parent **reciprocal function** is $y = \frac{1}{x}$.

Example

In an inverse variation, $x = 9$ when $y = 2$. What is the value of y when $x = 3$?

$2 = \frac{k}{9}$ .... Substitute 9 and 2 for x and y.

$18 = k$ .... Solve for k.

$y = \frac{18}{3}$ .... Substitute 18 and 3 for k and x, respectively.

$y = 6$ .... Divide.

Practice & Problem Solving

9. In an inverse variation, $x = 2$ when $y = -4$. What is the value of y when $x = 16$?
10. In an inverse variation, $x = 6$ when $y = \frac{1}{12}$. What is the value of x when $y = 2$?
11. Graph the function $y = \frac{5}{x}$. What are the domain, range, and asymptotes of the function?
12. **Look for Relationships** How is the parent reciprocal function related to an inverse variation?
13. **Make Sense and Persevere** The volume, V, of a gas varies inversely with pressure, P. If the volume of a gas is 6 cm^3 with pressure 25 kg/cm^2, what is the volume of a gas with pressure 15 kg/cm^2?

LESSON 4-2 Graphing Rational Functions

Quick Review

Vertical asymptotes may occur when the denominator of a rational function is equal to 0.

Horizontal asymptotes guide the end behavior of a graph and depend on the degrees of the numerator and denominator.

Example

What is the graph of $f(x) = \frac{9x^2 - 25}{x^2 - 5x - 6}$?

Find vertical asymptotes.

$x^2 - 5x - 6 = 0$ .... Set denominator equal to 0.

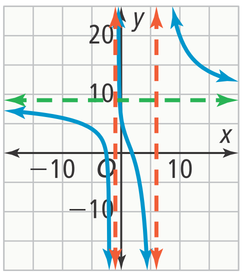
$(x + 1)(x - 6) = 0$ .... Factor.

$x = -1 \qquad x = 6$ .... Solve.

Find horizontal asymptotes.

Find the ratio of leading terms.

$f(x) = \frac{9x^2}{x^2} = 9$



Practice & Problem Solving

Identify the vertical and horizontal asymptotes of each rational function.

14. $f(x) = \frac{x - 8}{x^2 + 9x + 14}$

15. $f(x) = \frac{2x + 1}{x^2 + 5x - 6}$

16. $f(x) = \frac{x^2 - 9}{2x^2 + 25}$

17. $f(x) = \frac{16x^2 - 1}{x^2 - 6x - 16}$

Graph each function and identify the horizontal and vertical asymptotes.

18. $f(x) = \frac{x}{x^2 - 1}$

19. $f(x) = \frac{3}{x - 2}$

20. $f(x) = \frac{2x^2 + 7}{x^2 + 2x + 1}$

21. $f(x) = \frac{3x^2 - 11x - 4}{4x^2 - 25}$

22. Reason The daily attendance at an amusement park after day x is given by the function $f(x) = \frac{3{,}000x}{x^2 - 1}$. On approximately which day will the attendance be 1,125 people?

LESSON 4-3 Multiplying and Dividing Rational Expressions

Quick Review

To multiply **rational expressions**, divide out common factors and simplify. To divide rational expressions, multiply by the reciprocal of the divisor.

Example

What is the quotient of $\frac{x^2 + x - 2}{x + 3}$ and $\frac{x^2 + 3x - 4}{2x + 6}$?

$= \frac{x^2 + x - 2}{x + 3} \cdot \frac{2x + 6}{x^2 + 3x - 4}$ .... Multiply by reciprocal.

$= \frac{(x + 2)\cancel{(x - 1)}}{\cancel{x + 3}} \cdot \frac{2\cancel{(x + 3)}}{(x + 4)\cancel{(x - 1)}}$ .... Divide out common factors.

$= \frac{2(x + 2)}{x + 4}$ .... Simplify.

Practice & Problem Solving

Find the simplified product, and state the domain.

23. $\frac{x^2 + x - 12}{x^2 - x - 6} \cdot \frac{x + 2}{x + 4}$

24. $\frac{x^2 + 8x}{x^3 + 5x^2 - 24x} \cdot (x^3 + 2x^2 - 15x)$

Find the simplified quotient, and state the domain.

25. $\frac{x^2 - 36}{x^2 - 3x - 18} \div \frac{x^2 + 2x - 24}{x^2 + 7x + 12}$

26. $\frac{2x^2 + 5x - 3}{x^2 - 4x - 21} \div \frac{2x^2 + 5x - 3}{3x + 9}$

27. Reason The volume, in cubic units, of a rectangular prism with a square base can be represented by $25x^3 + 200x^2$. The height, in units, can be represented by $x + 8$. What is the side length of the base of the rectangular prism, in units?

LESSON 4-4 Adding and Subtracting Rational Expressions

Quick Review

To add or subtract rational expressions, multiply each expression in both the numerator and denominator by a common denominator. Add or subtract the numerators. Then simplify.

Example

What is the sum of $\frac{x-2}{x^2-25}$ and $\frac{3}{x+5}$?

$\frac{x-2}{(x+5)(x-5)} + \frac{3}{x+5}$ .... Factor denominators.

$= \frac{x-2}{(x+5)(x-5)} + \frac{3(x-5)}{(x+5)(x-5)}$ .... Find common denominator.

$= \frac{x-2+3(x-5)}{(x+5)(x-5)}$ .... Add numerators.

$= \frac{x-2+3x-15}{(x+5)(x-5)}$ .... Multiply.

$= \frac{4x-17}{(x+5)(x-5)}$ .... Simplify.

Practice & Problem Solving

Find the sum or difference.

28. $\frac{2x}{x+6} + \frac{3}{x-1}$

29. $\frac{x}{x^2-4} - \frac{5}{x-2}$

Simplify.

30. $\frac{2+\frac{2}{x}}{2-\frac{2}{x}}$

31. $\frac{\frac{-1}{x}+\frac{3}{y}}{\frac{4}{x}-\frac{5}{y}}$

32. Communicate Precisely Why is it necessary to consider the domain when adding and subtracting rational expressions?

33. Make Sense and Persevere Mia paddles a kayak 6 miles downstream at a rate 4 mph faster than the river's current. She then travels 6 miles back upstream at a rate 2 mph faster than the river's current. Write and simplify an expression for the time it takes her to make the round trip in terms of the river's current *c*.

LESSON 4-5 Solving Rational Equations

Quick Review

A **rational equation** is an equation relating rational expressions. An **extraneous solution** is a value that is a solution to an equation that is derived from an original equation but does not satisfy the original equation.

Example

What are the solutions to the equation $\frac{2}{x-2} = \frac{x}{x-2} - \frac{x}{4}$?

$(4)(x-2)\left(\frac{2}{x-2}\right)$

$= \left(\frac{x}{x-2} - \frac{x}{4}\right)(4)(x-2)$ .... Multiply by the LCD.

$8 = 4x - x^2 + 2x$ .... Multiply.

$x^2 - 6x + 8 = 0$ .... Write in standard form.

$(x-2)(x-4) = 0$ .... Factor.

$x-2=0$ or $x-4=0$ .... Zero Product Property

$x=2$ or $x=4$ .... Solve to identify possible solutions.

The solution $x = 2$ is extraneous. The only solution to the equation is $x = 4$.

Practice & Problem Solving

Solve the equation.

34. $\frac{18}{x+4} = 6$

35. $\frac{9}{x-1} = 3$

36. $-\frac{4}{3} + \frac{2}{x} = 8$

37. $\frac{2x}{x+3} = 5 + \frac{6x}{x+3}$

38. $-8 + \frac{64}{x-8} = \frac{x^2}{x-8}$

39. $\frac{9}{x^2-9} = \frac{3}{6(x-3)}$

40. Communicate Precisely Explain how to check if a solution to a rational equation is an extraneous solution.

41. Reason Diego and Stacy can paint a doghouse in 5 hours when working together. Diego works twice as fast as Stacy. Let *x* be the number of hours it would take Diego to paint the doghouse and *y* be the number of hours it would take Stacy to paint the doghouse. How long would it take Stacy to paint the doghouse if she was working alone? How long would it take Diego to paint the doghouse if he was working alone?

TOPIC 5

Topic Review

TOPIC ESSENTIAL QUESTION

1. How are rational exponents and radical equations used to solve real-world problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. In the expression $\sqrt[n]{c}$, n is the ________.
3. In the expression $\sqrt[n]{c}$, c is the ________.
4. Radicals with the same index and the same radicand are ________.
5. A(n) ________ is a function defined by a radical expression.
6. A(n) ________ of a number c is x, such that $x^n = c$.
7. A(n) ________ is a potential solution that must be rejected because it does not satisfy the original equation.
8. When all nth roots of perfect nth powers have been simplified and no radicals remain in the denominator, an expression is in ________.
9. A(n) ________ results from the application of one function to the output of another function.

- composite function
- extraneous solution
- index
- inverse function
- like radicals
- nth root
- radical function
- radicand
- reduced radical form

Concepts & Skills Review

LESSON 5-1 *n*th Roots, Radicals, and Rational Exponents

Quick Review

An ***n*th root** of a number c is x, such that $x^n = c$. The nth root of c can be represented as $\sqrt[n]{c}$, where n is the **index** and c is the **radicand**.

Example

Solve the equation $2x^4 = 162$.

$2x^4 = 162$ ········ Write the original equation.

$x^4 = 81$ ········ Divide both sides by 2.

$(x^4)^{\frac{1}{4}} = (81)^{\frac{1}{4}}$ ········ Raise both sides to the reciprocal of the exponent of x.

$x = \pm 3$ ········ Use the Power of a Power Property.

Practice & Problem Solving

What is the value of each expression? Round to the nearest hundredth, if necessary.

10. $\sqrt[4]{16^2}$

11. $-\sqrt[3]{25^6}$

Simplify each expression.

12. $\sqrt[3]{27x^{12}}$

13. $\sqrt[4]{16a^{24}b^8}$

Solve each equation.

14. $750 = 6y^3$

15. $1{,}280 = 5z^4$

16. **Communicate Precisely** Describe the relationship between a rational exponent and a root of a number x.

17. **Make Sense and Persevere** The function $d(t) = 9.8t^2$ represents how far an object falls, in meters, in t seconds. How long would it take a rock to fall from a height of 300 m? Round to the nearest hundredth of a second.

LESSON 5-2 Properties of Exponents and Radicals

Quick Review

To simplify radical expressions, look for factors that are perfect *n*th power factors.

The Product Property of Radicals and Quotient Property of Radicals can also be used to rewrite radical expressions.

Product Property of Radicals $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example

What is $\sqrt[4]{64} \cdot \sqrt[4]{2}$ in reduced radical form?

$\sqrt[4]{64} \cdot \sqrt[4]{2}$ .... Write the original expression.

$\sqrt[4]{64 \cdot 2}$ .... Use the Product Property of Radicals.

$\sqrt[4]{128}$ .... Multiply.

$\sqrt[4]{16} \cdot \sqrt[4]{8}$ .... Rewrite using the Product Property of Radicals.

$2\sqrt[4]{8}$ .... Simplify.

Practice & Problem Solving

What is the reduced radical form of each expression?

18. $\sqrt{x^6y^4} \cdot \sqrt{x^8y^6}$

19. $\sqrt[3]{\frac{243m^4}{3m}}$

20. $\sqrt[3]{5x^4} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{25x^3}$

21. $\sqrt{\frac{98a^{10}}{2a^4}}$

Multiply.

22. $(\sqrt{n} - \sqrt{7})(\sqrt{n} + 3\sqrt{7})$

23. $(9x + \sqrt{2})(9x + \sqrt{2})$

24. $(5\sqrt{3} + 6)(5\sqrt{3} - 6)$

25. $\sqrt[3]{4}\,(6\sqrt[3]{2} - 1)$

How can you rewrite each expression so there are no radicals in the denominator?

26. $\frac{6}{1 + \sqrt{2}}$

27. $\frac{5}{2 - \sqrt{5}}$

28. $\frac{4 + \sqrt{6}}{3 - 3\sqrt{6}}$

29. $\frac{-9x}{\sqrt{x}}$

30. Error Analysis Describe and correct the error made in rewriting the radical expression.

$5\sqrt{18} - \sqrt{27} = 7\sqrt{2}$

31. Reason A rectangular wall is $\sqrt{240}$ ft by $\sqrt{50}$ ft. You need to paint the wall twice to cover the area with two coats of paint. If each can of paint can cover 60 square feet, how many cans of paint will you need?

LESSON 5-3 Graphing Radical Functions

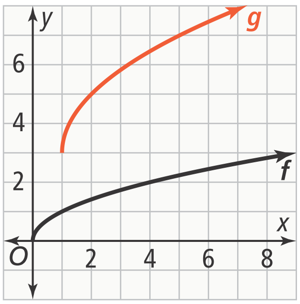
Quick Review

A **radical function** is a function defined by a radical expression. To determine transformations of a radical function, write the radical function in the form $h(x) = a\sqrt[n]{x-h} + k$ and compare it to the parent function.

Example

Graph $g(x) = 2\sqrt{x-1} + 3$.

$g(x) = 2\sqrt{x-1} + 3$ is a vertical stretch by a factor of 2, a horizontal shift 1 unit to the right, and a vertical shift 3 units up from the parent function $f(x) = \sqrt{x}$.



Practice & Problem Solving

Graph the following functions. What are the domain and range? Is the function increasing or decreasing?

32. $f(x) = \sqrt{x} - 1$

33. $f(x) = \sqrt[3]{x} + 2$

34. $f(x) = \frac{1}{2}\sqrt{x+1}$

35. $f(x) = 2\sqrt[3]{x} - 1$

36. $f(x) = \sqrt[3]{x-3}$

37. $f(x) = \sqrt{x+4} - 2$

38. Communicate Precisely Explain how to rewrite the function $g(x) = \sqrt[3]{8x-24} + 1$ to identify the transformations from the parent graph $f(x) = \sqrt[3]{x}$.

39. Reason The speed s, in miles per hour, of a car when it starts to skid can be estimated using the formula $s = \sqrt{30 \cdot 0.5d}$, where d is the length of the skid marks, in feet. Graph the function. If a car's skid marks measure 40 ft in a zone where the speed limit is 25 mph, was the car speeding? Explain.

LESSON 5-4 Solving Radical Equations

Quick Review

To solve a radical equation, isolate the radical. Raise both sides of the equation to the appropriate power to eliminate the radical and solve for x. Then check for **extraneous solutions**. If the equation includes more than one radical, eliminate one radical at a time using a similar process.

Example

Solve the radical equation $\sqrt{6-x} = x$.

$\sqrt{6-x} = x$ ······ Write the original equation.

$(\sqrt{6-x})^2 = (x)^2$ ······ Square both sides.

$6 - x = x^2$ ······ Simplify.

$0 = x^2 + x - 6$ ······ Write in standard form.

$0 = (x+3)(x-2)$ ······ Factor.

$x = -3$ or $x = 2$ ······ Use the Zero-Product Property.

Check the solutions to see if they both make the original equation true.

Practice & Problem Solving

Solve each radical equation. Check for extraneous solutions.

40. $\sqrt[3]{x} - 2 = 7$

41. $\sqrt{2x} = 12$

42. $\sqrt{25+x} + 5 = 9$

43. $13 - \sqrt[4]{x} = 10$

44. $\sqrt{5x+1} + 1 = x$

45. $\sqrt{6x-20} - x = -6$

46. Construct Arguments Give an example of a radical equation that has no real solutions. Explain your reasoning.

47. Make Sense and Persevere The formula $d = \frac{\sqrt{15w}}{3.14}$ gives the diameter d, in inches, of a rope needed to lift a weight of w, in tons. How much weight can be lifted with a rope that has a diameter of 4 in?

LESSON 5-5 Function Operations

Quick Review

You can add, subtract, multiply, or divide functions. When adding and subtracting functions, the domain is the intersection of the domains of the two functions. When multiplying and dividing functions, the domain is the set of all real numbers for which both original functions and the new function are defined. You can also compose functions, by using one function as the input for another function. These are called **composite functions**.

Example

Let $f(x) = 5x$ and $g(x) = 3x - 1$. What is the rule for the composition $f \circ g$?

$f \circ g = f(g(x))$ ······ Apply the definition.

$= f(3x - 1)$ ······ Apply the rule for g.

$= 5(3x - 1)$ ······ Apply the rule for f.

$= 15x - 5$ ······ Distribute.

Practice & Problem Solving

Let $f(x) = -x + 6$ and $g(x) = 5x$. Identify the rule for the following functions.

48. $f + g$

49. $f - g$

50. $g(f(2))$

51. $f(g(-1))$

52. Reason For the functions f and g, what is the domain of $f \circ g$? $\frac{f}{g}$? $\frac{g}{f}$?

53. Make Sense and Persevere A test has a bonus problem. If you get the bonus problem correct, you will receive 2 bonus points and your test score will increase by 3% of your score. Let $f(x) = x + 2$ and $g(x) = 1.03x$, where x is the test score without the bonus problem. Find $g(f(78))$. What does $g(f(78))$ represent?

LESSON 5-6 Inverse Relations and Functions

Quick Review

An **inverse relation** is formed when the roles of the independent and dependent variables are reversed. If an inverse relation of a function, f, is itself a function, it is called the **inverse function** of f, which is written $f^{-1}(x)$.

Example

What is the inverse of the relation represented in the table?

x	y
–2	0
–1	6
0	5
1	3
3	–1

Switch the values of x and y. Then reorder the ordered pairs.

x	y
–1	3
0	–2
3	1
5	0
6	–1

Practice & Problem Solving

Find an equation of the inverse function.

54. $f(x) = -4x^2 + 3$

55. $f(x) = \sqrt{x - 4}$

56. $f(x) = 9x + 5$

57. $f(x) = \sqrt{x + 7} - 1$

58. Error Analysis Jamie said the inverse of $f(x) = \sqrt{x - 9}$ is $f^{-1}(x) = (x + 9)^2$. Is Jamie correct? Explain.

59. Make Sense and Persevere An electrician charges $50 for a house visit plus $40 per hour. Write a function for the cost C of an electrician charging for h hours. Find the inverse of the function. If the bill is $150, how long did the electrician work?

TOPIC 6

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use exponential and logarithmic functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A(n) ________ has base e.
3. A(n) ________ has the form $f(x) = a \cdot b^x$.
4. In an exponential function, when $0 < b < 1$, b is a(n) ________.
5. The ________ allows logarithms with a base other than 10 or e to be evaluated.
6. A(n) ________ has base 10.
7. The inverse of an exponential function is a(n) ________.

- decay factor
- exponential function
- logarithmic function
- growth factor
- common logarithm
- natural logarithm
- Change of Base Formula

Concepts & Skills Review

LESSON 6-1 Key Features of Exponential Functions

Quick Review

An **exponential function** has the form $f(x) = a \cdot b^x$. When $a > 0$ and $b > 1$, the function is an **exponential growth function**. When $a > 0$ and $0 < b < 1$, the function is an **exponential decay function**.

Example

Paul invests $4,000 in an account that pays 2.5% interest annually. How much money will be in the account after 5 years?

Write and use the exponential growth function model.

$A(t) = a(1 + r)^n$

$A(5) = 4{,}000(1 + 0.025)^5$

$A(5) = 4{,}000(1.025)^5$

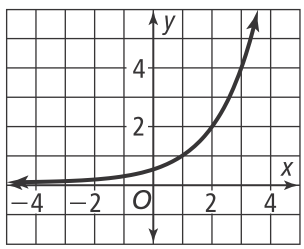
$A(5) = 4{,}525.63$

There will be about $4,525.63 in Paul's account after 5 years.

Practice & Problem Solving

Identify the domain, range, intercept, and asymptote of each exponential function. Then describe the end behavior.

8. $f(x) = 400 \cdot \left(\frac{1}{2}\right)^x$
9. $f(x) = 2 \cdot (3)^x$
10. **Reason** Seth invests $1,400 at 1.8% annual interest for 6 years. How much will Seth have at the end of the sixth year?
11. **Model With Mathematics** Bailey buys a car for $25,000. The car depreciates in value 18% per year. How much will the car be worth after 3 years?
12. Identify the domain, range, intercept, and asymptote.



TOPIC 6 REVIEW

Quick Review

Interest may be compounded over different time periods, such as quarterly, monthly, or daily. The formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ is used to calculate the amount of money available after it has been invested for an amount of time. Interest may also be compounded continuously. The formula $A = Pe^{rt}$ is used to calculate the amount of money available in an account that is compounded continuously. The calculator can be used to find an exponential model for a set of data.

Example

Jenny invests $2,500 in an account that pays 2.4% interest annually. The interest is compounded quarterly. How much will Jenny have in the account after 6 years?

Use the formula $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$.

$A = 2{,}500\left(1 + \frac{0.024}{4}\right)^{4(6)}$ ······ Substitute for *A, P, n,* and *r*.

$A = 2{,}500(1.006)^{24}$ ······ Simplify.

$A = 2{,}885.97$ ······ Use a calculator.

Jenny will have about $2,885.97.

Practice & Problem Solving

Find the total amount of money in the account after the given amount of time.

13. Compounded quarterly, $P = \$12{,}000$, $r = 3.6\%$, $t = 4$ years

14. Compounded monthly, $P = \$5{,}000$, $r = 2.4\%$, $t = 8$ years

15. Continuously compounded, $P = \$7{,}500$, $r = 1.6\%$, $t = 10$ years

Write an exponential model given two points.

16. (12, 256) and (13, 302)

17. (3, 54) and (4, 74)

18. **Model With Mathematics** Jason's parents invested some money for Jason's education when Jason was born. The table shows how the account has grown.

Number of Years	Amount ($)
1	2,250
3	2,525
6	3,480
7	4,400
9	6,000
13	9,250

Predict how much will be in the account after 18 years.

LESSON 6-3 Logarithms

Quick Review

A logarithm is an exponent. Common logarithms have base 10 and natural logarithms have base *e*. Exponential expressions can be rewritten in logarithmic form, and logarithmic expressions can be converted to exponential form.

$5^3 = 125$ can be rewritten as $\log_5 125 = 3$.

$\log 100 = 2$ can be rewritten as $10^2 = 100$.

Example

Evaluate $\log_2 \frac{1}{8}$.

$\log_2 \frac{1}{8} = x$ ········ Write an equation.

$2^x = \frac{1}{8}$ ········ Rewrite the equation in exponential form.

$2^x = 2^{-3}$ ········ Rewrite the equation with a common base.

$x = -3$ ········ Since the two expressions have a common base, the exponents are equal.

Practice & Problem Solving

Use Structure If an equation is given in exponential form, write the logarithmic form. If an equation is given in logarithmic form, write the exponential form.

19. $4^3 = 64$

20. $10^2 = 100$

21. $\log_6 216 = 3$

22. $\ln 20 = x$

Evaluate each logarithmic expression.

23. $\log_8 \frac{1}{64}$

24. $\log_4 81$

Use Appropriate Tools Evaluate each logarithmic expression using a calculator. Round answers to the nearest thousandth.

25. $\log 628$

26. $\ln 0.55$

Evaluate each logarithmic expression.

27. $\log_5 5^9$

28. $7^{\log_7 49}$

LESSON 6-4 Logarithmic Functions

Quick Review

A logarithmic function is the inverse of an exponential function.

Example

Find the inverse of $f(x) = 10^{x-2}$. Identify any intercepts or asymptotes.

$y = 10^{x-2}$ ········ Write in $y = f(x)$ form.

$x = 10^{y-2}$ ········ Interchange x and y.

$y - 2 = \log x$ ········ Write in log form.

$y = \log x + 2$ ········ Solve for y.

The equation of the inverse is $f^{-1}(x) = \log x + 2$. It has an x-intercept at $x = \frac{1}{100}$ and a vertical asymptote at the y-axis.

Practice & Problem Solving

Look for Relationships Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.

29. $f(x) = \log_4 x$

30. $f(x) = \ln x - 2$

Use Structure Find the equation of the inverse of each function.

31. $f(x) = 8^{x-2}$

32. $f(x) = \frac{5^{x-2}}{8}$

LESSON 6-5 Properties of Logarithms

Quick Review

Properties of logarithms can be used to either expand a single logarithmic expression into individual logarithms or condense several logarithmic expressions into a single logarithm.

The Change of Base Formula can be used to find logarithms of numbers with bases other than 10 or *e*.

Example

Use the properties of logarithms to expand the expression $\log_6 \frac{x^3 y^5}{z}$.

$\log_6 \frac{x^3 y^5}{z}$

$= \log_6 x^3y^5 - \log_6 z$ ········ Quotient Property of Logarithms

$= \log_6 x^3 + \log_6 y^5 - \log_6 z$ ···· Product Property of Logarithms

$= 3\log_6 x + 5\log_6 y - \log_6 z$ ···· Power Property of Logarithms

Practice & Problem Solving

Use Structure **Use the properties of logarithms to write each as a single logarithm.**

33. $3\log r - 2\log s + \log t$

34. $2\ln 3 + 4\ln 2 - \ln 36$

Evaluate each logarithm.

35. $\log_4 12$

36. $\log_7 70$

Make Sense and Persevere **Use the Change of Base Formula to solve each equation for *x*. Give an exact solution written as a logarithm and an approximate solution rounded to the nearest thousandth.**

37. $5^x = 200$

38. $7^x = 486$

LESSON 6-6 Exponential and Logarithmic Equations

Quick Review

You can solve exponential equations by taking the logarithm of both sides. You can solve a logarithmic equation by combining the logarithmic terms into one logarithm and then converting to exponential form.

Example

Solve $7^{2x} = 10^{x+1}$.

$7^{2x} = 10^{x+1}$

$\log 7^{2x} = \log 10^{x+1}$ ········ Take the common log of each side.

$2x \log 7 = (x + 1) \log 10$ ···· Power Property of Logarithms

$2x \log 7 = x + 1$ ········ Since $\log 10 = 1$

$2x \log 7 - x = 1$ ········ Subtract *x* from each side.

$x(2 \log 7 - 1) = 1$ ········ Factor out *x*.

$x = \frac{1}{2 \log 7 - 1}$ ········ Divide each side by $2 \log 7 - 1$.

$x \approx 1.449$ ········ Use a calculator.

Practice & Problem Solving

Find all solutions of the equation. Round answers to the nearest ten-thousandth.

39. $2^{5x+1} = 8^{x-1}$

40. $9^{2x+3} = 27^{x+2}$

41. $3^{x-2} = 5^{x-1}$

42. $7^{x+1} = 12^{x-1}$

Find all solutions of the equation. Round answers to the nearest ten thousandth

43. $\log_5 (3x - 2)^4 = 8$

44. $\ln (x^2 - 32) = \ln (4x)$

45. $\log_6 (2x - 1) = 2 - \log_6 x$

46. **Model With Mathematics** Geri has $1,500 to invest. He has a goal to have $3,000 in this investment in 10 years. At what annual rate, compounded continuously, will Geri reach his goal? Round the answer to the nearest thousandth.

LESSON 6-7 Geometric Sequences and Series

Quick Review

A geometric sequence is defined by a common ratio between consecutive terms. It can be defined explicitly or recursively. A geometric series is the sum of the terms of a geometric sequence.

Example

A geometric sequence is defined by $a_n = \begin{cases} \frac{1}{9}, & n = 1 \\ 3a_{n-1}, & n > 1 \end{cases}$. What is the sum of the first 10 terms of this sequence?

$a_n = \frac{1}{9}(3)^{n-1}$ Write the explicit definition.

$a_{10} = \frac{1}{9}(3)^9 = 2187$ Find the 10th term.

$S_{10} = \dfrac{\frac{1}{9}(1 - 3^{10})}{(1 - 3)} = 3280\frac{4}{9}$ Calculate the sum.

Practice & Problem Solving

Determine whether or not each sequence is geometric.

47. 2, 4, 6, 8, 10, 12, …

48. 2, 4, 8, 16, 32, 64, …

Convert between recursive and explicit forms.

49. $a_n = \begin{cases} \frac{1}{8}, & n = 1 \\ \frac{3}{2}a_{n-1}, & n > 1 \end{cases}$

50. $a_n = -2(5)^{n-1}$

Find the sum for each geometric series.

51. $\sum_{k=1}^{8} 3(2)^k$

52. $\sum_{k=1}^{9} 243\left(\frac{1}{3}\right)^k$

53. **Look for Relationships** Find the difference $\sum_{k=1}^{10} 5(2)^k - \sum_{k=1}^{10} 10(2)^k$. Explain how you found your answer.

54. **Make Sense and Persevere** The half-life of carbon-14 is 5,730 years. This is the amount of time it takes for half of a sample to decay. From a sample of 24 grams of carbon 14, how long will it take until only 3 grams of the sample remains?

TOPIC 6 REVIEW

TOPIC 7

Topic Review

TOPIC ESSENTIAL QUESTION

1. How are trigonometric functions used to solve real-world problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ of an angle in standard position is along the positive x-axis.

3. The __________ of an angle is the other side of an angle in standard position.

4. The distance between the midline and the minimum or maximum of a periodic function is called the __________.

5. The __________ of a periodic function is the reciprocal of the period.

6. A horizontal translation of a periodic function is often called a __________.

7. If an angle θ is in standard position, the __________ for θ is the acute angle formed by the x-axis and the terminal side of θ.

- amplitude
- frequency
- initial side
- phase shift
- reference angle
- terminal side

Concepts & Skills Review

LESSON 7-1 Trigonometric Functions and Acute Angles

Quick Review

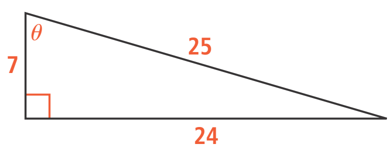
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\quad\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\quad\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\quad\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Example

Write the six trigonometric ratios for the angle θ in the given triangle.



$\sin\theta = \dfrac{24}{25}$ $\quad\csc\theta = \dfrac{25}{24}$

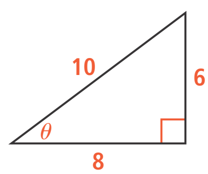
$\cos\theta = \dfrac{7}{25}$ $\quad\sec\theta = \dfrac{25}{7}$

$\tan\theta = \dfrac{24}{7}$ $\quad\cot\theta = \dfrac{7}{24}$

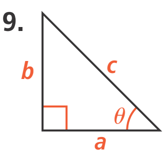
Practice & Problem Solving

Write the six trigonometric ratios for the angle θ in each given triangle.

8.



9.



What are the trigonometric ratios of θ in a right triangle with the given value?

10. $\sin\theta = \dfrac{5}{13}$

11. $\cot\theta = \dfrac{56}{33}$

12. Look for Relationships What trigonometric ratio is given by the cofunction identity $\sec(90^\circ - \theta)$?

13. Make Sense and Persevere A 15-foot ladder is leaning against the side of a house at a 65° angle. What is the distance from the house to the base of the ladder? Round to the nearest hundredth.

LESSON 7-2 Angles and the Unit Circle

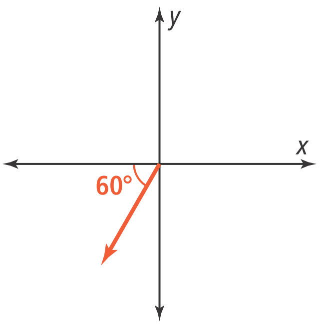
Quick Review

An angle is in **standard position** when its vertex is at the origin and the initial side lies on the *x*-axis. Angles in standard position may be named with positive values or negative values.

The **unit circle** is a circle that has its center at the origin and has a radius of 1. An angle of full circle rotation, or 360°, has a measure of 2π radians.

Example

What is the measure of this angle as a positive number of degrees and in radians? As a negative number of degrees and in radians?



$$m\angle\theta = 180° + 60° = 240°$$
$$= 240°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{3}$$
$$m\angle\theta = 240° - 360° = -120°$$
$$= 120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$$

Practice & Problem Solving

Find a positive angle measure for each reference angle.

14. 67° in Quadrant I

15. 63° in Quadrant IV

16. 25° in Quadrant II

17. 14° in Quadrant III

Convert the angle measures.

18. 136° to radians

19. $\frac{2\pi}{3}$ radians to degrees

20. 80° to radians

21. $-\frac{\pi}{3}$ radians to degrees

For each angle give the reference angle and Quadrant.

22. $-\frac{3\pi}{4}$ radians

23. 330°

24. **Communicate Precisely** Why is it convenient to express an angle in radians when you want to compute arc length?

25. **Model With Mathematics** The radius of a pond is about 840 feet. After walking around the pond through an angle of $\frac{2\pi}{3}$, you pick up a plastic bottle. You carry it to a recycle bin at a point where you have walked through an angle of $\frac{5\pi}{4}$. How far did you carry the bottle?

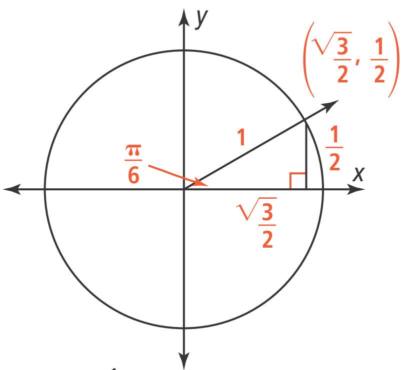
LESSON 7-3 Trigonometric Functions and Real Numbers

Quick Review

The domains of the sine and cosine functions are extended to all real numbers using the unit circle. The coordinates of the point where the terminal side of an angle in standard position intersects the unit circle are $(\cos\theta, \sin\theta)$. The values of the other trigonometric functions can be calculated from this result.

Example

Use the unit circle to evaluate $\tan\frac{\pi}{6}$.



$$\tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \bullet \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Practice & Problem Solving

Find the sine and cosine for each angle.

26. $\frac{4\pi}{3}$

27. $135°$

28. $\frac{5\pi}{6}$

29. $420°$

Find the tangent for each angle.

30. $120°$

31. $-\frac{\pi}{4}$

Find the secant, cosecant, and cotangent for each angle.

32. $-135°$

33. $\frac{8\pi}{3}$

34. **Use Structure** What is $\sin\theta$ if $\cos\theta = \frac{3}{5}$ and θ is in Quadrant IV?

35. **Reason** A scout team is searching a circular region in a 6-mile radius around a camp. Two of the scouts travel on a route that is 45° east of south from the camp. What is their final position, relative to the camp?

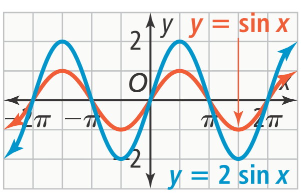
LESSON 7-4 Graphing Sine and Cosine Functions

Quick Review

The distance between the midline and the minimum or maximum point is the **amplitude**. The **period** is the interval of the domain for which the function does not repeat. **Frequency** is the reciprocal of the period.

Example

What are the amplitude, period, and frequency of $y = 2\sin x$?



The distance between the midline and maximum point is 2, so the amplitude is 2. The period is 2π. The frequency is 1.

Practice & Problem Solving

What are the amplitude, period, and frequency of each function?

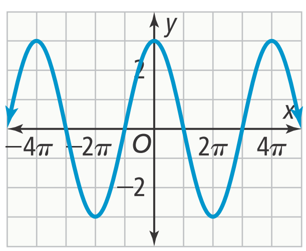
36. $y = \frac{1}{4}\cos(4x)$

37. $y = 3\sin\left(\frac{1}{2}\right)x$

38. $y = 4\sin 2x$

39. $y = -2\cos 6x$

40. **Use Structure** What equation represents the graph?



LESSON 7-5 Graphing Other Trigonometric Functions

Quick Review

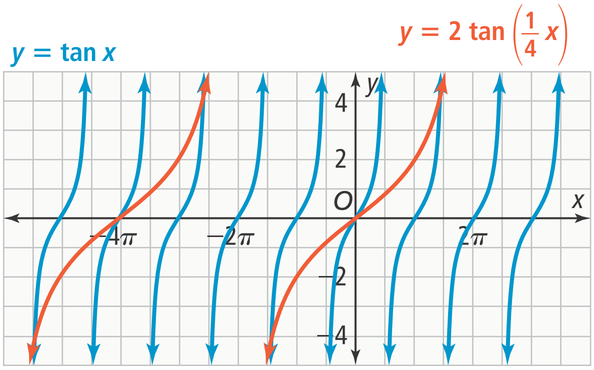
When graphing $y = a \tan bx$, a stretches the graph of the parent function vertically and b compresses the graph of the parent function horizontally. The period of the tangent function can be found using period $= \frac{\pi}{|b|}$.

Example

How can you use transformations of the parent function to sketch the graph of the function $y = 2 \tan \frac{1}{4}x$?

$a = 2$, so stretch the graph vertically by a factor of 2.

$b = \frac{1}{4}$, so stretch the graph horizontally by a factor of 4.



Practice & Problem Solving

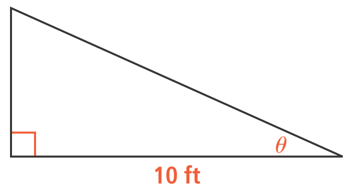
Sketch the graph of the function. Then describe how the parent graph of the function was affected by the transformations.

41. $y = \frac{1}{4} \tan 2x$

42. $y = \frac{1}{2} \cot 3x$

43. **Use Structure** Describe the domain, range, period, zeros, and asymptotes of the function $y = \cot x$.

44. **Reason** Write a function that represents the height, h, of the triangle where θ is the angle indicated. Graph the function over the domain $[0, \frac{\pi}{2}]$.



45. **Make Sense and Persevere** The function $y = 3 \sec \theta$ models the length of a pole leaning against a wall as a function of the measure of the angle θ formed by the pole and the horizontal when the bottom of the pole is 3 ft from the wall. Graph the function and find the length the pole when $\theta = 62°$. Round to the nearest hundredth.

LESSON 7-6 Translating Trigonometric Functions

Quick Review

A horizontal translation of a periodic function is the **phase shift**. When graphing $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, $|a|$ is the amplitude, $\frac{|b|}{2\pi}$ is the frequency, c is the phase shift, and d is the vertical shift.

Example

What are the key features of the function $y = 5 \cos 2(x - 1) + 6$.

$a = 5$, so the amplitude is 5.

$b = 2$, so the frequency is 2, which means the period is $\frac{2\pi}{2} = \pi$.

$c = 1$, so the phase shift is 1 unit to the right.

$d = 6$, so the vertical shift is 6 units up.

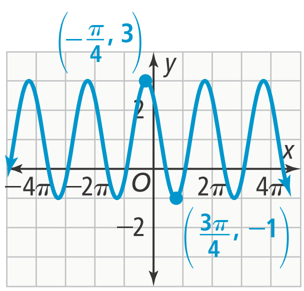
Practice & Problem Solving

Identify the amplitude, period, phase shift, and vertical shift of the function.

46. $y = -4 \sin (x + 4\pi) - 8$

47. $y = \frac{1}{4} \cos\left[6\left(x + \frac{\pi}{2}\right)\right] + 2$

48. **Use Structure** Write an equation that models the function represented by the graph using the cosine function.



TOPIC 7 REVIEW

TOPIC 8

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do trigonometric identities and equations help you solve problems involving real or complex numbers?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A(n) __________ is a trigonometric equation that is true for all values of the variable for which both sides of the equation are defined.
3. The __________ has two axes, like the Cartesian coordinate plane, which are the real axis and the imaginary axis.
4. The horizontal axis, also called the __________, is for the real part of a complex number.
5. The vertical axis, also called the __________, is for the imaginary part of a complex number.
6. The __________ is the length of the segment from the point that corresponds to the complex number to the origin.
7. The angle θ measured counterclockwise from the positive real axis to the segment is the __________.

- Law of Sines
- Law of Cosines
- trigonometric identity
- complex plane
- real axis
- imaginary axis
- modulus of a complex number
- argument
- polar form of a complex number

Concepts & Skills Review

LESSON 8-1 Solving Trigonometric Equations Using Inverses

Quick Review

An inverse trigonometric function allows you to input the values in a limited range of a trigonometric function and find the corresponding measure of an angle in the domain of the trigonometric function.

Example

Solve the trigonometric equation $5 \sin \theta = 3 \sin \theta + 1$ for values between 0 and 2π.

$5 \sin \theta = 3 \sin \theta + 1$ Write the original equation.

$2 \sin \theta = 1$ Subtract $3 \sin \theta$.

$\sin \theta = \frac{1}{2}$ Divide by 2.

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ Find the sine inverse.

$\theta = \frac{\pi}{6}$ Solve.

Reflect the angle $\frac{\pi}{6}$ across the y-axis; that angle will also have a sine of $\frac{1}{2}$. That angle is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Practice & Problem Solving

Evaluate each function. Angle values must be within the range of each inverse function. Give answers in radians and in degrees.

8. $\tan^{-1}(\sqrt{3})$
9. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$
10. $\tan^{-1}(-1)$
11. $\cos^{-1}\left(-\frac{1}{2}\right)$

Solve each trigonometric equation for values between 0 and 2π.

12. $3 \tan x - \sqrt{3} = 0$
13. $2 \cos x + \sqrt{2} = 0$

14. **Reason** Why is the domain of the inverse sine function restricted to the interval $[-1, 1]$?

15. **Make Sense and Persevere** A bird flies 78 ft from the top of a 4 ft tall bird feeder to the top of a 65 ft tree. To the nearest degree, find the angle of elevation of the line of sight from the top of the bird feeder to the top of the tree.

TOPIC 8 REVIEW

LESSON 8-2 Law of Sines and Law of Cosines

Quick Review

The Law of Sines and the Law of Cosines allow you to apply trigonometric functions to non-right triangles.

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $a^2 = b^2 + c^2 - 2bc(\cos A)$

Example

In $\triangle ABC$, $m\angle A = 93°$, $a = 15$, and $b = 11$. To the nearest degree, what is $m\angle B$?

$\frac{\sin A}{a} = \frac{\sin B}{b}$ ······ Use the Law of Sines.

$\frac{\sin 93}{15} = \frac{\sin B}{11}$ ······ Substitute.

$\frac{11 \sin 93}{15} = \sin B$ ······ Isolate the sine function.

$\sin^{-1}\left(\frac{11 \sin 93}{15}\right) = B$ ······ Use the inverse sine function.

$m\angle B \approx 47°$ ······ Solve.

Practice & Problem Solving

Use the Law of Sines to solve.

16. In $\triangle MNP$, $m\angle N = 112°$, $n = 14$, and $p = 6$. What is $m\angle P$?

17. In $\triangle XYZ$, $m\angle X = 40°$, $m\angle Y = 25°$, and $x = 13$. What is y?

In $\triangle QRS$, find $m\angle Q$.

18. $q = 7$, $r = 6$, $s = 10$

19. $q = 8$, $r = 5$, $s = 6$

20. **Look for Relationships** How do you know whether to use the Law of Sines or the Law of Cosines to solve a problem?

21. **Make Sense and Persevere** Mark went to the beach, parked his car, and walked 500 yd down a path toward the beach. Mark then turned onto a boardwalk at a 125° angle to his path and walked another 140 yd along the boardwalk to a pier. If Mark turns to face his car, what angle does he turn?

LESSON 8-3 Trigonometric Identities

Quick Review

Quotient Identity: $\tan x = \frac{\sin x}{\cos x}$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

Cofunction Identities: $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

Odd-Even Identities: $\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

Sum and Difference Formulas:

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$

Example

What is the simplified form of $\frac{\csc^2 x - 1}{\csc^2 x}$?

$\frac{\csc^2 x - 1}{\csc^2 x} = \frac{\csc^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$ ······ Rewrite the fraction.

$= 1 - \sin^2 x$ ······ Use the definition of sine.

$= \cos^2 x$ ······ Apply the Pythagorean Identity.

Practice & Problem Solving

Use a trigonometric identity to write a different form of each expression.

22. $\tan^2 x + 1$

23. $\tan x + \cot x$

24. $\frac{1 + \tan^2 x}{1 - \tan^2 x}$

25. $\frac{\sec x - 1}{\sec x + 1}$

Find the exact value of each expression. Then evaluate the function on your calculator. Compare the calculator value to your exact value.

26. $\sin 15°$

27. $\cos 105°$

28. $\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

29. $\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

30. **Use Structure** Find expressions for $\sin 2\theta$ and $\cos 2\theta$.

31. **Model With Mathematics** Is a noise with a sound wave modeled by $y = \sin(1{,}500\pi x)$ cancelled out by another noise with a sound wave modeled by $y = \sin\left[1{,}500\pi\left(x - \frac{1}{250}\right)\right]$? Explain.

LESSON 8-4 The Complex Plane

Quick Review

A complex number has the form $a + bi$, where a and b are real numbers. The **complex plane** has two axes. The horizontal axis, or **real axis,** is for the real part of a complex number. The vertical axis, or **imaginary axis,** is for the imaginary part of a complex number. The **modulus of a complex number** is the distance from the point representing the complex number to the origin. The *distance* between two complex numbers is the modulus of the *difference* between the two numbers.

Example

What is the modulus of 4 – 3*i*?

$$|z| = \sqrt{z \bullet \bar{z}}$$
$$= \sqrt{(4 + 3i)(4 - 3i)}$$
$$= \sqrt{16 - 9i^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

Practice & Problem Solving

Find the midpoint of the segment that joins the complex numbers.

32. $-7 + 5i,\ 3 - 15i$

33. $1 + 9i,\ 11 - i$

Find the modulus of each complex number.

34. $13 + 8i$

35. $-3 + i$

Find the distance between the complex numbers.

36. $r = -14 + 4i,\ s = -8 + i$

37. $r = 7 + 16i,\ s = -3 - 13i$

38. **Look for Relationships** What is the relationship between the modulus of a complex number and the modulus of its complex conjugate? Explain.

39. **Make Sense and Persevere** On a coordinate plane, a library is located at the coordinates $(-5, 9i)$. The fire station is located at $(7, -7i)$. The school is halfway between the library and the fire station. What are the coordinates of the school?

LESSON 8-5 Polar Form of Complex Numbers

Quick Review

Rectangular form of a complex number: $z = a + bi$
Polar form of a complex number: $z = r \text{ cis } \theta$

$$r = |z| = \sqrt{a^2 + b^2}$$

$a = r\cos\theta$ $\quad$ $b = r\sin\theta$

Convert to polar form: $\tan\theta = \frac{b}{a}$ so $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Convert to rectangular form: $a = r\cos\theta$, $b = r\sin\theta$

Product Formula: $(r \text{ cis } \alpha)(s \text{ cis } \beta) = rs \text{ cis}(\alpha + \beta)$

Power Formula: $z^n = r^n \text{ cis } n\theta$

Example

Express $z = 4 \text{ cis } \frac{\pi}{3}$ in rectangular form.

$a = r\cos\theta$	$b = r\sin\theta$
$a = 4\cos\frac{\pi}{3}$	$b = 4\sin\frac{\pi}{3}$
$a = 2$	$b = 2\sqrt{3}$

$z = a + bi = 2 + 2i\sqrt{3}$

Practice & Problem Solving

Write each expression in rectangular form.

40. $z = 3 \text{ cis } \frac{\pi}{6}$

41. $z = 6 \text{ cis } \frac{2\pi}{3}$

Write each expression in polar form.

42. $z = 3 + 5i$

43. $z = -1 - 2i$

Find the product of z_1z_2.

44. $z_1 = 1 + i\sqrt{2}$ and $z_2 = \sqrt{2} - i$

45. Find z^3 where $z = -2 - 3i$ in rectangular form.

46. **Reason** What complex number can you square to get $4 \text{ cis } \frac{\pi}{2}$? Explain.

47. **Use Structure** Determine the voltage in a circuit when there is a current of $3 \text{ cis } \frac{\pi}{4}$ amps and an impedance of $2 \text{ cis } \frac{2\pi}{3}$ ohms. (*Hint*: Use $E = I \bullet Z$, where E is voltage, I is current, and Z is impedance.)