PRACTICE & PROBLEM SOLVING

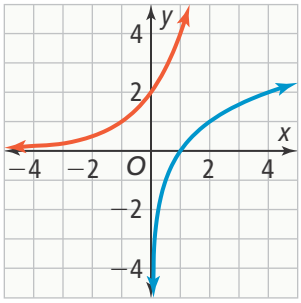
Scan for Multimedia

Practice Tutorial

Additional Exercises Available Online

UNDERSTAND

7. Look for Relationships Are the logarithmic and exponential functions shown inverses of each other? Explain.



8. Communicate Precisely How is the graph of the logarithmic function $g(x) = \log_2 (x - 7)$ related to the graph of the function $f(x) = \log_2 x$? Explain your reasoning.

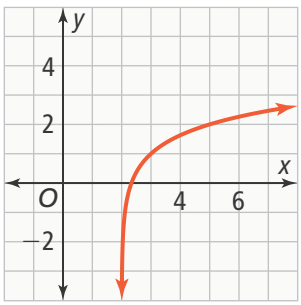
9. Error Analysis Describe and correct the error a student made in finding the inverse of the exponential function $f(x) = 5^{x-6} + 2$.

$y = 5^{x-6} + 2$	Write in $y = f(x)$ form.
$x = 5^{y-6} + 2$	Interchange x and y.
$x - 2 = 5^{y-6}$	Subtract 2 from each side.
$y - 6 = \log_5 x - 2$	Rewrite in logarithmic form.
$y = \log_5 x - 2 + 6$	Add 6 to each side.
$y = \log_5 x + 4$	Simplify.
$f'(x) = \log_5 x + 4$	

✗

10. Make Sense and Persevere The number of members m who joined a new workout center w weeks after opening is modeled by the equation $m = 1.6^{w+2}$, where $0 \le w \le 10$. Find the inverse of the function and explain what the inverse tells you.

11. Use Structure The graph shows a transformation of the parent graph $f(x) = \log_3 x$. Write an equation for the graph.



PRACTICE

Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior. SEE EXAMPLE 1

12. $y = \log_5 x$

13. $y = \log_8 x$

14. $y = \log_{\frac{3}{10}} x$

15. $y = \log_{0.1} x$

Describe the graph in terms of transformations of the parent function $f(x) = \log_6 x$. Compare the asymptote and x-intercept of the given function to the parent function. SEE EXAMPLE 2

16. $g(x) = \frac{1}{2} \log_6 x$

17. $g(x) = \log_6 (-x)$

18. Describe how the graph of $g(x) = -\ln(x + 0.5)$ is related to the graph of $f(x) = \ln x$. SEE EXAMPLE 2

Find the equation of the inverse of each function. SEE EXAMPLE 3

19. $f(x) = 5^{x-3}$

20. $f(x) = \left(\frac{1}{2}\right)^{x-1}$

21. $f(x) = 6^{x+7}$

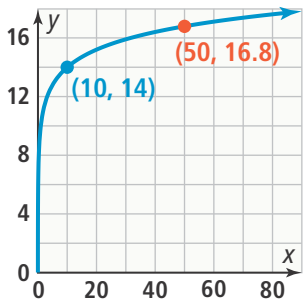
22. $f(x) = \log_2 (8x)$

23. $f(x) = \ln (x + 3) - 1$

24. $f(x) = 4 \log_2 (x - 3) + 2$

25. The altitude y, in feet, of a plane t minutes after takeoff is approximated by the function $y = 5{,}000 \ln(.05t) + 8{,}000$. Solve for t in terms of y. What is a situation in which it would be easier to use your new equation rather than the original? SEE EXAMPLE 4

26. Find the average rate of change of the function graphed below over the interval $10 \le x \le 50$. Compare it to the average rate of change of $y = 3 \log x + 12$ over the same interval. SEE EXAMPLE 5



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APPLY

27. Model with Mathematics The equation $r = 90 - 25 \log(t + 1)$ is to model a student's retention r after taking a physics course where r represents a student's test score (as a percent), and t represents the number of months since taking the course.

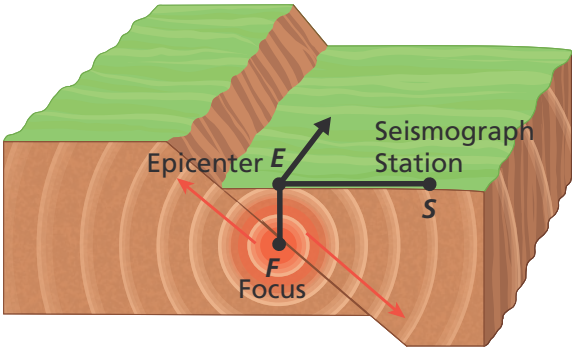
a. Make a table of values for ordered pairs that represent $r = 90 - 25 \log(t + 1)$, rounding to the nearest tenth. Then sketch the graph of the function on a coordinate plane through those ordered pairs. (You may use a graphing calculator to check.)

b. Find the equation of the inverse. Interpret the meaning of this function.

28. Higher Order Thinking As shown by the diagram, an earthquake occurs below Earth's surface at point F (the focus). Point E, on the surface above the focus, is called the *epicenter*. A seismograph station at point S records the waves of energy generated by the earthquake. The surface wave magnitude M of the earthquake is given by this formula:

$$M = \log\left(\frac{A}{T}\right) + 1.66(\log D) + 3.3$$

In the formula, A is the amplitude of the ground motion in micrometers, T is the period in seconds, and D is the measure of ES in degrees.



a. Find surface wave magnitude of an earthquake with $A = 700$ micrometers, $T = 2$ and $D = 100°$.

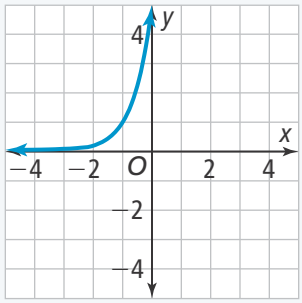
b. In the formula, $20° < D \leq 160°$. By how much can the size of arc ES affect the surface wave magnitude? Explain.

ASSESSMENT PRACTICE

29. The logarithmic function $g(x) = \ln x$ is transformed to $h(x) = \ln(x + 2) - 1$. Which of the following are true? Select **all** that apply.

Ⓐ $g(x)$ is translated 2 units upward.
Ⓑ $g(x)$ is translated 2 units to the right.
Ⓒ $g(x)$ is translated 2 units to the left.
Ⓓ $g(x)$ is translated 1 unit downward.
Ⓔ $g(x)$ is translated 1 unit to the left.
Ⓕ The vertical asymptote shifts 2 units to the left.
Ⓖ The vertical asymptote shifts 2 units to the right.

30. SAT/ACT The graph shows the exponential function $f(x) = 5^{x+1}$. Which of the following functions represents its inverse, $f^{-1}(x)$?



Ⓐ $f^{-1}(x) = 1 + \log_5 x$ Ⓒ $f^{-1}(x) = \log_5 (x - 1)$

Ⓑ $f^{-1}(x) = \log_5 x - 1$ Ⓓ $f^{-1}(x) = \log_5 (x + 1)$

31. Performance Task The logarithmic function $M(d) = 5 \log d + 2$ is used to find the limiting magnitude of a telescope, where d represents the diameter of the lens of the telescope (mm) that is being used for the observation.

Part A Find the limiting magnitude of a telescope having a lens diameter of 40 mm.

Part B Find the equation of the inverse of this function.

Part C Interpret why astronomers may wish to use the inverse of this function. Justify your reasoning.

Part D Using the inverse function, find the diameter of the lens that has a limiting magnitude of 13.5. Check your answer with the table function of your graphing calculator.