PRACTICE & PROBLEM SOLVING

UNDERSTAND

12. Generalize Explain the process that is used to verify that a trigonometric equation is an identity.

13. Construct Arguments Benjamin said that he had worked out a trigonometric identity: $\cos 2\theta = \cos^2\theta - \sin^2\theta$. Is Benjamin correct? Explain.

14. Mathematical Connections Show that $f(x) = \tan x$ is an odd function by verifying that $\tan(-x) = -\tan x$.

15. Error Analysis Describe and correct the error a student made in applying the cosine difference formula to find the exact value of cos 15°.

$$\begin{aligned}\cos 15^\circ &= \cos(45^\circ - 30^\circ)\\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ\\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

16. Construct Arguments Show that the quotient identity $\cot\theta = \frac{\cos\theta}{\sin\theta}$ is true algebraically.

17. Higher Order Thinking Use the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$ to algebraically derive each of the following identities.

a. $1 + \tan^2 x = \sec^2 x$

b. $1 + \cot^2 x = \csc^2 x$

18. Look for Relationships Using the Pythagorean Identity, express $\sin\theta$ in terms of $\cos\theta$.

19. Use Structure Restate the Cofunction Identities using degrees instead of radians. What can you conclude about sin 15°? cos 60°?

PRACTICE

20. Does the relationship between $\csc(-\theta)$ and $-\csc\theta$ indicate whether $\csc\theta$ is odd or even? SEE EXAMPLE 1

21. Does the relationship between $\sec(-\theta)$ and $-\sec\theta$ indicate whether $\sec\theta$ is odd or even? SEE EXAMPLE 1

22. Does the relationship between $\cot(-\theta)$ and $-\cot\theta$ indicate whether $\cot\theta$ is odd or even? SEE EXAMPLE 1

Find a simplified form of each expression. SEE EXAMPLE 2

23. $\dfrac{\cos\theta}{\sin\theta\cot\theta}$

24. $[\cot(-x)](\sin x)$

Prove each of the following sum and difference formulas. SEE EXAMPLE 3

25. $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

26. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Find the exact value of each expression. Then evaluate the function on your calculator, comparing the calculator value to the approximation for your exact value. SEE EXAMPLE 4

27. tan 105°

28. $\sin\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$

29. cos 225°

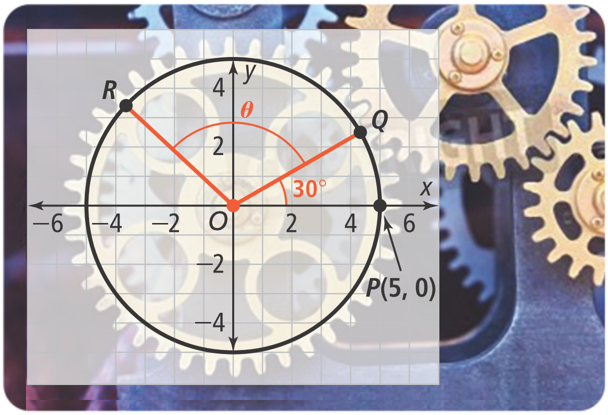
30. sin 75°

31. Does a noise with a sound wave modeled by $y = \sin(660\pi x)$ get cancelled out by another noise with a sound wave modeled by $y = \sin\left[660\pi\left(x - \frac{1}{220}\right)\right]$? Explain. SEE EXAMPLE 5

32. The sound wave for a musical note is modeled by $y = \sin(1{,}320\pi x)$. The sound wave for a different note is modeled by $y = \sin\left[1{,}320\pi\left(x + \frac{1}{220}\right)\right]$. What is the simplified form of an equation that models the sound wave if the two notes are played simultaneously? SEE EXAMPLE 5

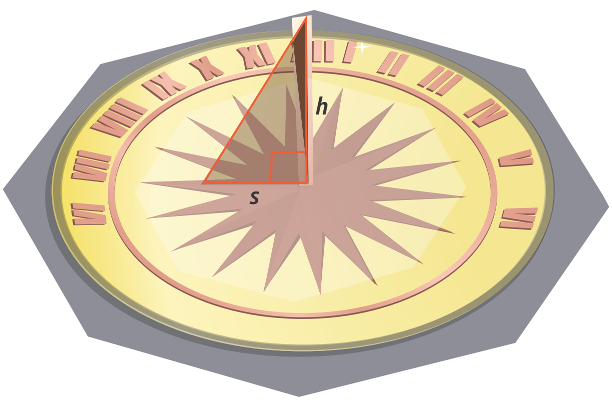
APPLY

33. Make Sense and Persevere The diagram shows a gear with a radius of 5 in. Point Q represents a 30° counterclockwise rotation of point $P(5, 0)$. Point R represents a further θ-degree rotation. The coordinates of R are $(5\cos(\theta + 30°), 5\sin(\theta + 30°))$. Express these coordinates in terms of $\cos\theta$ and $\sin\theta$.



34. Look for Relationships The force required to push an object at a certain angle from its resting position can be modeled by $F = Mg\tan\theta$, where F is the force, M is the mass of the object, g is the acceleration due to gravity, and θ is the angle at which the object is being pushed. Write an equivalent equation for this formula in terms of $\sin\theta$ and $\sec\theta$.

35. Reason The length s of a shadow cast by a vertical *gnomon* (the column or shaft on a sundial that projects a shadow) of height h when the angle of the sun above the horizon is θ can be modeled by the equation $s = \frac{h\sin(90° - \theta)}{\sin\theta}$. Show that this equation is equivalent to $s = h\cot\theta$. (*Hint*: Convert degrees to radians and use a cofunction identity).



ASSESSMENT PRACTICE

36. Fill in each blank to complete an expression equivalent to $\frac{\csc x(\sin^2 x + \cos^2 x\tan x)}{\sin x + \cos x}$.

a. $\sin^2 x +$ ________

b. ____________ $- \cos^2 x$

37. SAT/ACT Find the exact value of $\tan 75°$.

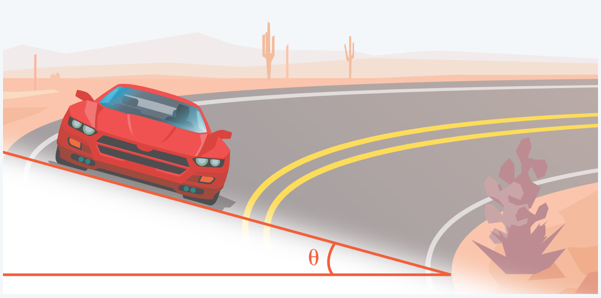
Ⓐ $2 + \sqrt{3}$

Ⓑ $2 - \sqrt{3}$

Ⓒ $-2 + \sqrt{3}$

Ⓓ $-2 - \sqrt{3}$

38. Performance Task In order for motor vehicles to negotiate a curve in the road without skidding or running off of it, the angle of incline of the road must be determined. The angle of incline, or *angle of inclination,* is the nonnegative acute angle that the vehicle makes with the horizontal, and it is represented by the equation $\tan\theta = \frac{v^2}{gR}$, where R is the radius of the circular path, v is the speed that the vehicle is traveling in meters per second, and g is the acceleration due to gravity, 9.8 m/s^2.



Part A Find the angle of inclination of a curve with a 120-m radius, when a vehicle is traveling at 60 km/h (16.7 m/s) around the curve. Round to the nearest tenth of a degree.

Part B Write an equivalent equation in terms of $\sin\theta$, rather than $\tan\theta$.