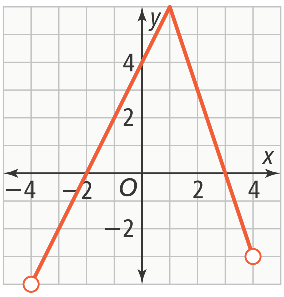
Selected Answers

Topic 1

Lesson 1-1

2. The zeros of a function are the input values that result in a function output value of 0. **4.** (–4, 4] **6.** $x = -3$, $x = -1$, $x = 4$ **8.** (–4, –3) and (–1, 4) **10.** (–2, 2) **12.** There is a zero at $x = 4$ because the function is changing from negative to positive at $x = 4$.

14.

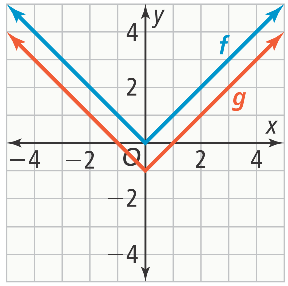


16. A relative maximum occurs when a function changes from increasing to decreasing. A relative minimum occurs when a function changes from decreasing to increasing. **18.** domain: (–∞, ∞); range: [–9, ∞) **20.** negative: (–4, 2); positive: (–∞, –4) and (2, ∞) **22.** 1 **24.** *x*-intercepts: –5, 3; *y*-intercept: 1.5 **26.** increasing: (–5, –1); decreasing: (–1, 5) **28.** domain: [0, 80]; range: [0, 100]; increasing: [0, 80]; decreasing: none; *x*-intercept: 0; *y*-intercept: 0; positive: (0, 80]; negative: none **30. a.** $f(x) = 100 - 0.5x$ **b.** domain: [0, 200]; range: [0, 100] **c.** 3 hours 20 minutes **32.** A

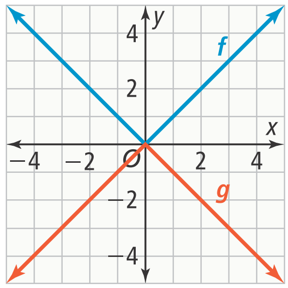
Lesson 1-2

2. $g(x) = f(x) + k$ is a vertical translation k units, so the output of the function is affected. $g(x) = f(x - h)$ is a horizontal translation h units, so the input of the function is affected.

4.



6.



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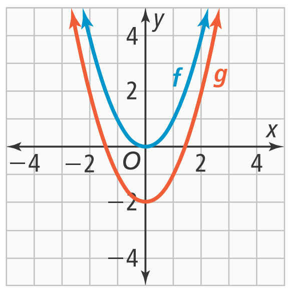

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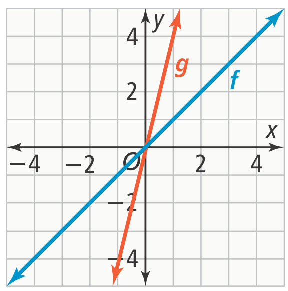
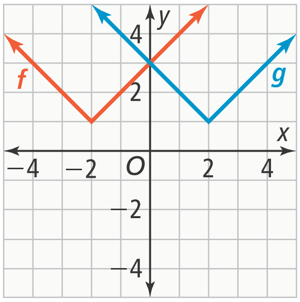


12. Vertical and horizontal translations and reflections do not change the shape of a graph. Stretches and compressions change the shape of a graph.

Selected Answers

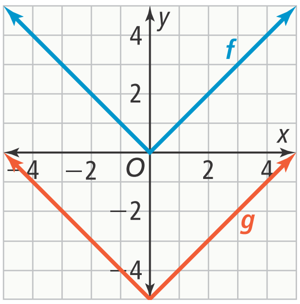
Topic 1

14. The student reflected the graph of $f(x)$ across the x-axis instead of reflecting the graph across the y-axis.



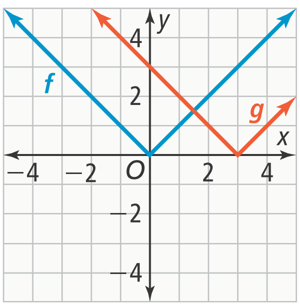
16. $g(x) = f(-x) = (-x)^2 = (x)^2$; The graph of $g(x)$ is the same as the graph of $f(x)$.

18.



The domain values of f and g are the same, but the range values of g are 5 units less than the range values of f.

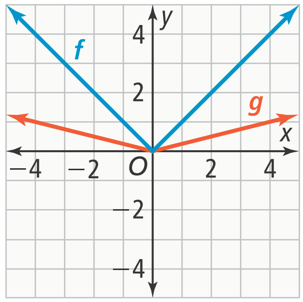
20.



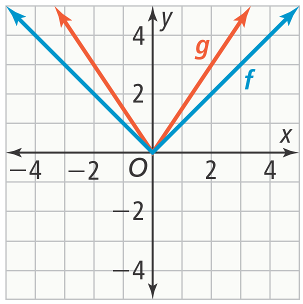
The range values of f and g are the same, but the domain values of g are 3 units greater than the domain values of f at corresponding range values.

22. $f(x) = -x^2 - 1$

24.



26.



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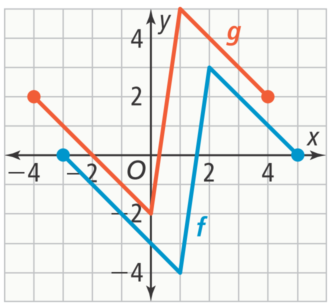
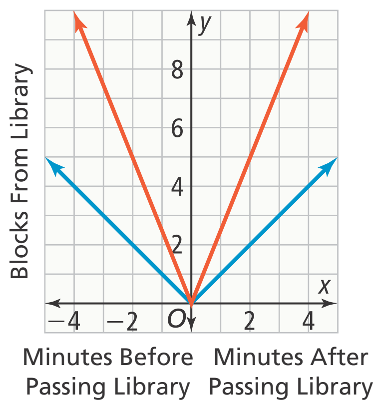


30. translation right 3 units and then translation up 5 units **32.** translation right 7 units, translation down 9 units, and then vertical stretch by a factor of 4 **34. a.** $f(x) = |x|$ **b.** $g(x) = 2.5|x|$

c.



Selected Answers

Topic 1

36. A. $y = |x| - 1$ D. $y = |x + 1| - 1$

B. $y = -|x + 1|$ E. $y = 2|x| - 1$

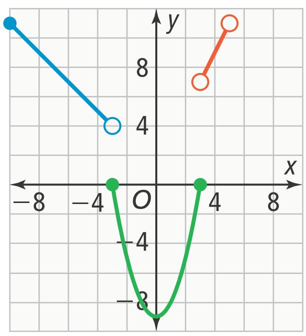
C. $y = -|x + 1| - 1$ F. $y = |x + 1|$

38. Part A $g(x) = -1.22|x - 17.7| + 21.6$ **Part B** domain: $0 \le x \le 35.4$; range: $0 \le y \le 21.6$; The domain represents the width of the pyramid, which is 35.4 m wide. The range represents the height of the pyramid, which is 21.6 m high.

Lesson 1-3

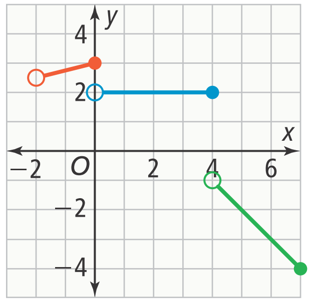
2. Piecewise functions are two or more functions over different intervals, while step functions are two or more constant values for different intervals. **4.** Divide the graph into the sections defined by the domain. Graph the function given in each section. Note whether the symbols of the domain require an open or closed circle at the edges of each function graph.

6.



8. decreasing: –2 **10.** The graph has open circles at $x = 3$, so the domain is all real numbers except 3.
12. $-2 < x \le 3$ and $4 \le x < 9$
14. step function

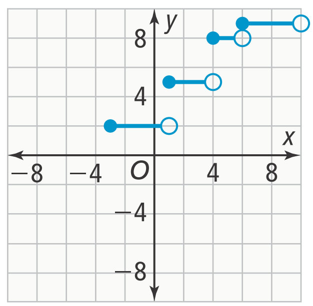
16.



domain: $-2 < x \le 7$; range: $-4 \le y < -1$ and $y = 2$ and $2.5 < y \le 3$; increasing: between –2 and 0; constant: between 0 and 4; decreasing: between 4 and 7

18. $f(x) = \begin{cases} 3x + 1, & \text{if } x \ge -\frac{1}{3} \\ -3x - 1, & \text{if } x < -\frac{1}{3} \end{cases}$

20.



22. $P(h) = \begin{cases} 10h, \text{ when } 0 < h \le 40 \\ 15h - 200, \text{ when } h > 40 \end{cases}$; \$475 **24.** domain: $\{x \mid 0 < x \le 80\}$ range: $\{y \mid 12.25, 16.75, 19.50, 23.50, 25.25\}$ **26.** A

Lesson 1-4

2. An arithmetic sequence is a set of numbers with a common difference between consecutive terms. An arithmetic series is the sum of the numbers in an arithmetic sequence. **4.** Evaluate $2n + 1$ for $n = 1, 2, 3, 4, 5$, and then add all five of those answers together for the final sum. **6.** –5; –19, –24, –29 **8.** –1; –8, –9, –10

Selected Answers

Topic 1

10. 8; 23, 31, 39 **12.** Answers may vary. Sample: 17, 11, 5, –1, …
Explicit formula: $a_n = 17 - 6(n - 1)$
Recursive formula: $a_n = \begin{cases} 17, \text{ if } n = 1 \\ a_{n-1} - 6, \text{ if } n > 1 \end{cases}$
14. at least 9 sales **16.** 440
18. yes; 10; 50 **20.** no
22. $a_n = \begin{cases} 2, \text{ if } n = 1 \\ a_{n-1} + 2, \text{ if } n > 1; \end{cases}$ $a_n = 2 + 2(n - 1)$
24. $a_n = \begin{cases} 0, \text{ if } n = 1 \\ a_{n-1} + \frac{1}{8}, \text{ if } n > 1; \end{cases}$ $a_n = \frac{1}{8}(n - 1)$
26. a. $a_n = 1 + 2(n - 1)$ **b.** 15 members
28. 1,095 **30.** –69 **32.** 105 tiles
34. a. No; there will be a total of only 450 laptops. **b.** Cons: Ones purchased in the first year may break before all students have one. Pros: Spread out the cost. **36.** in the sequence: 60, 39, 81; not in the sequence: 68, 75
38. Part A $a_n = 175 + 28(n - 1)$
Part B 724.5 g **Part C** 31 wk

Lesson 1-5

2. Solving an equation graphically by finding points of intersection can be helpful when the equations are complicated or impossible to solve algebraically. Also it can be useful when estimating solutions. **4.** $x = 3$; The x-coordinate of the intersection point is 3. **6.** Victor incorrectly said the solution is the y-value of one of the points of intersection instead of the x-value. One solution is $x \approx 1.47$.

8. For $x = 1.1426$, $x = 1.1427$, and $x = 1.1428$, $f(x) < g(x)$. For $x = 1.1429$ and $x = 1.1430$, $g(x) < f(x)$. So, the solution of $f(x) = g(x)$ is between $x = 1.1428$ and $x = 1.1429$. **10.** $x = 0$ and $x = 16$ **12.** $x = 4$ **14.** $x = -3$
16. $x > 3$ **18.** $-4 \le x \le 2$ **20.** $x > 4$
22. $x \approx 0.228$ and $x \approx 8.772$ **24.** $x \approx -0.869$ and $x \approx 1.535$ **26.** $x \approx -8.4$ and $x \approx 1.5$ **28.** $x \approx -6.2$ and $x \approx 7.2$
30. a. $7.75x > 5.5x + 2.45$ **b.** Zhang will be ahead of Jack after about 1.09 h.
32. a. $0 < (-2x^2 + 18x - 2) - (-0.25x + 6)$, or $0 < -2x^2 + 18.25x - 8$
b. between 462 and 8,663 items **34.** E

Lesson 1-6

2. No; Savannah switched the x- and y-values. The solution is (2, –1).
4. Knowing how to solve a system of two equations helps you understand the process of eliminating a variable and solving for another variable. This helps in solving a system of three equations. **6.** (3, 2)

8. $\begin{cases} x - 2y = 2 \\ -4x + 3y = -5 \end{cases}$

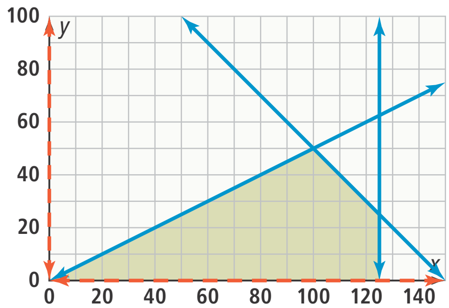
10. Each row represents an equation in the system when written in standard form. **12.** When the determinant is equal to zero, the two equations have the same slope, so they represent either parallel lines or the same line. Therefore, there is either no solution or an infinite number of solutions.

14. $\begin{cases} x \ge -4 \\ x \le 4 \\ y \ge -4 \\ y \le 4 \end{cases}$ **16.** (3, 4) **18.** (4, 1)

Selected Answers

Topic 1

20.



22. infinite solutions **24.** $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -2 & 17 \end{array}\right]$

26. $\left[\begin{array}{cc|c} 10 & -5 & 3 \\ 2 & 1 & 0 \end{array}\right]$ **28.** $\begin{cases} 2x - 2y = 4 \\ x + 2y = 11 \end{cases}$

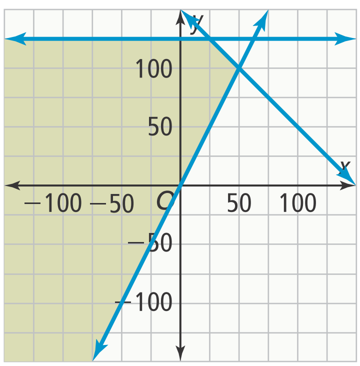
30. $\begin{cases} 0.10x + 0.25y = 1.25 \\ x + y = 8 \end{cases}$;

$\left[\begin{array}{cc|c} 0.10 & 0.25 & 1.25 \\ 1 & 1 & 8 \end{array}\right]$

32. 8 free throws, 6 two-point baskets, and 2 three-point baskets

34. a. $\begin{cases} y \geq 2x \\ y \leq 125 \\ 2x + 2y \leq 300 \end{cases}$

b.



c. Keisha can design a rectangular giraffe enclosure with length x and width y shown by the solution region in the graph. **36. a.** No **b.** Yes **c.** Yes **d.** Yes **e.** No **38.** Assume that at each game there are the same number of sophomores, juniors, and seniors.

Part A $\begin{cases} 3x + 4y + 4z = 2{,}319 \\ 4x + 4y + 3z = 2{,}290 \\ 5x + 6y + 7z = 3{,}785 \end{cases}$

Part B $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 185 \\ 0 & 1 & 0 & 227 \\ 0 & 0 & 1 & 214 \end{array}\right]$

Part C Check students' work.

Lesson 1-7

2. True; two equations are needed to solve a system that has two variables. Three equations are needed to solve a system that has three variables.
4. Write each row of the system in standard form. Then write a matrix to represent the system. Enter the matrix in MATRIX[A] in a graphing calculator. Use the rref function to show the matrix in reduced row echelon form. The values in the last column are the solution. **6.** (10, 15, 25)

8. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$ **10.** The row echelon matrix that results from a series of row operations can be unique, but is not always unique. There could be a different set of row operations that will produce the same row echelon matrix. The reduced row echelon matrix is unique because each matrix has only one reduced row echelon matrix.
12. Dylan did not write each row of the system in standard form before writing the matrix to represent the system.
14. A row will have zeros on the coefficient side of the matrix.
16. Answers may vary.

Sample: $\begin{cases} 2x + 3y = -5 \\ -x + 2y = -8 \end{cases}$

Selected Answers

Topic 1

18. (4, 2) **20.** $\left(-\frac{5}{2}, 2, 10\right)$ **22.** $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 2 \end{array}\right]$

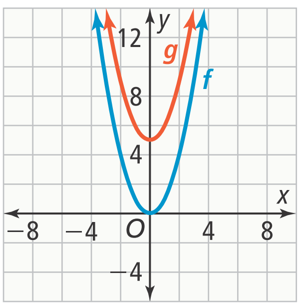
24. $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ **26.** no solution

28. 6 game downloads and 7 song downloads **30.** The craft store ordered twice as many markers as acrylic paints.
32. B

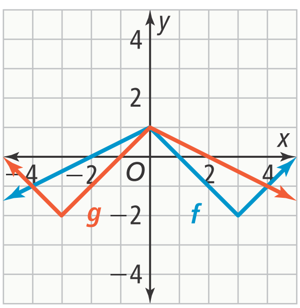
Topic Review

2. step function **4.** transformation
6. piecewise-defined function
8. domain: $(-5, \infty)$; range: $[-4, 1]$; zero: $x = 2$; positive: $(2, 5]$; negative: $(-5, 2)$
10. domain: $[0, 100]$; range: $[0, 50]$; increasing: none; decreasing: $[0, 100]$; x-intercept: 100; y-intercept: 50; positive: $[0, 100]$; negative: none

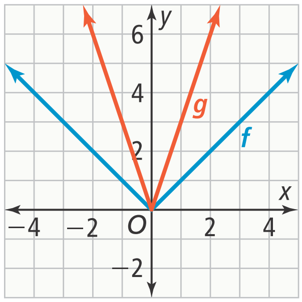
12.



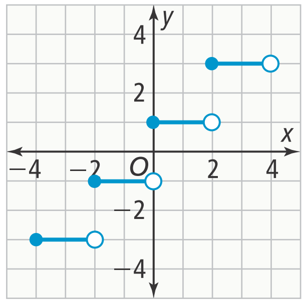
14.



16.



18.



20. $f(x) = \begin{cases} -\frac{3}{2}x - \frac{15}{2}, \text{ if } x < -3 \\ \frac{1}{5}x + \frac{8}{5}, \text{ if } -3 \le x \le 2 \\ 3, \text{ if } \; 2 < x \le 5 \end{cases}$

22. $C(n) = \begin{cases} 120n, \text{ when } 0 < n \le 10 \\ 90n + 1{,}200, \text{ when } n > 10; \$1{,}920 \end{cases}$

24. $d = -6$; -5

26. $a_n = \begin{cases} 25, \text{ if } n = 1 \\ a_{n-1} - 7, \text{ if } n > 1; \; a_n = 25 - 7(n - 1) \end{cases}$

28. 582 **30.** 93 **32.** $x = -2$ and $x = 1$
34. $(-\infty, -3)$ and $(1, \infty)$ **36.** No; Noninteger answers are not easily obtained from a graph. Sometimes you can only approximate the answer.
38. $x = -1$, $y = 3$ **40.** Answers may vary. Sample: $2x - y = 3$ and $4x - 2y = 6$

42. $\left[\begin{array}{cc|c} 4 & 8 & 12 \\ -2 & -6 & 32 \end{array}\right]$; $\left[\begin{array}{cc|c} 1 & 0 & 41 \\ 0 & 1 & -19 \end{array}\right]$

44. This maintains the relationship between the variables and the solutions while you perform row operations.

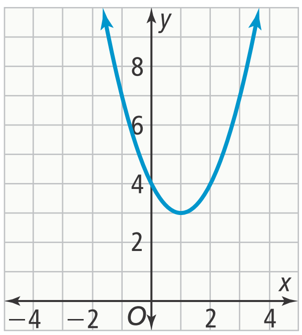
Selected Answers

Topic 2

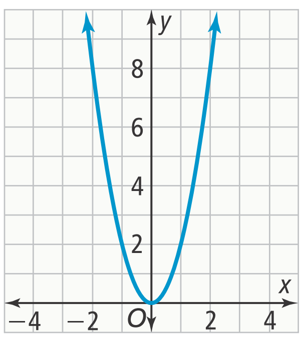
Lesson 2-1

2. Martin should rewrite the function as $g(x) = (x - (-3))^2$. Then he could see that h is really -3 and the graph should be translated 3 units left from the parent function. **4.** The graph of the function g is a reflection across the x-axis and a translation 2 units to the left and 4 units down of the graph of f. **6.** The graph of the function h is a translation 2 units to the left and 7 units down. **8.** $y = 2(x - 1)^2 + 3$ **10.** $y = 3(x - 7)^2 + 4$ **12.** No, the correct vertex is $(-2, -4)$. The classmate should rewrite the function as $g(x) = (x - (-2))^2 - 4$ to see the signs for the values of the vertex (h, k). **14.** Since $a = -1$ and $-1 < 0$, the parabola opens downward.

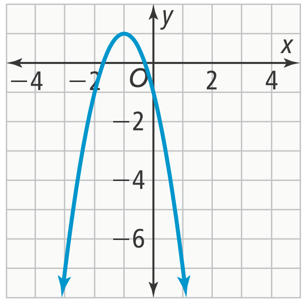
16. The graph of the function is translated 1 unit right and 3 units up from the graph of $f(x) = x^2$.



18. The function g is a vertical stretch of the graph of $f(x) = x^2$.



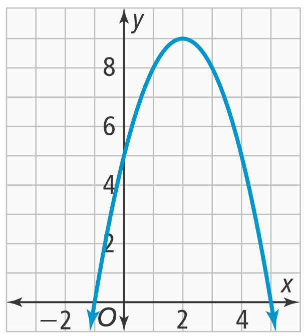
20. The graph of the function is a reflection in the x-axis, translated 1 unit left and 1 unit up with a vertical stretch of the graph of $f(x) = x^2$.



22. Vertex: $(2, 5)$; axis of symmetry: $x = 2$; minimum: $y = 5$; domain: $(-\infty, \infty)$; range: $[5, \infty)$ **24.** Vertex: $(-4, 0)$; axis of symmetry: $x = -4$; maximum: $y = 0$; domain: $(-\infty, \infty)$; range: $(-\infty, 0]$ **26.** $y = -7(x - 1)^2 + 2$ **28.** $y = -7x^2 + 5$ **30.** $g(x) = (x + 4)^2 + 1$ **32.** The maximum height for Max Jumps is 9 in. The maximum height for Jumpsters is 7 in. There is a 2-inch difference in the maximum heights. **34.** 105 m

36. no; yes; no; no; no; yes

38. Part A The domain is $\{x \mid x > 0\}$. The value of x cannot be negative since it represents the price of a cookie.



Part B $f(0.40) = \$398.75$; $f(0.75) = \$355$ **Part C** The bakery should charge \$0.45 per cookie to make the maximum profit. **Part D** The maximum profit the bakery can make is \$400.

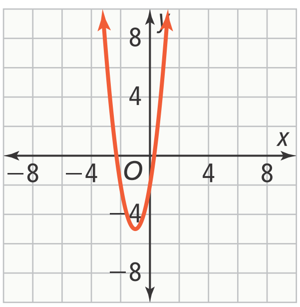
Selected Answers

Topic 2

Lesson 2-2

2. The y-intercept may or may not represent the maximum value. The y-intercept is the same as the y-coordinate of the vertex if the parabola opens downward and the vertex is on the y-axis. **4.** Since the standard form of the equation has 3 coefficients: a, b, and c, three ordered pairs are needed to determine the values of each coefficient. Solve the system of 3 equations to find the equation in standard form and then graph it. **6.** vertex: (2, 11); y-intercept: (0, 7) **8.** Minimum: –51

10.



12. No; This is a fairly accurate model of the data, but it is not the exact model found when substituting the 3 points into the standard equation and solving for a, b, and c. **14.** First find the vertex. Since the vertex is on the x-axis, the top of the bowl can be represented by a horizontal line with y equal to the bowl's depth. You can calculate the diameter by setting y equal to the depth and solving for x, then finding the difference between the two solutions. **16.** (3, 39)
18. vertex: (3, 1); y-intercept: (0, –8)
20. vertex: (–3, –17); y-intercept: (0, 10) **22.** 540 ft **24.** $y = 2x^2 + 4x + 2$
26. $y = -3.5x^2 + 3.35x + 0.7$

28. a. Equation of the parabola: $y = -x^2 + 24x$; Vertex form: $y = -(x - 12)^2 + 144$; Intercept form: $y = -x(x - 24)$
b. maximum area: 144 sq. in.; length: 12; width: 12 **30.** E

Lesson 2-3

2. Amir is not correct. There are no real solutions to the equation $y = x^2 + 16$.
4. The zeros of a function are the only points where a function can change from positive to negative or negative to positive, so you can separate a function into intervals separated by the zeros and any point within an interval will have the same sign.
6. $(5x - 2)(x + 1)$ **8.** $x = 2$, $x = -\frac{3}{4}$
10. Yes; if b and c are the zeros, the equation is $y = a(x - b)(x - c)$. To find the value of a, substitute 0 for x and the value for the y-intercept for y, and solve for a. Then multiply the factors. **12.** $y = (x + 1)(x + 6)$ **14.** The student is not correct; the zeros of a quadratic function in factored form are not simply the constants in the expression, but are found using the Zero Product Property, so $x - 2 = 0$ or $x = 2$, and $x + 7 = 0$ or $x = -7$.
16. When you factor an expression, the solution of each factor, when set equal to zero, yields the x-intercepts. **18.** $(3x + 4)(x - 3)$
20. $(2x - 3)(x + 5)$ **22.** $(4x + 1)(x - 3)$
24. –2, 7 **26.** –4, 5 **28.** $-\frac{7}{4}$, 1
30. 4 seconds **32.** $x < -4$, $x > 2$
34. $-2 < x < 6$ **36.** $x < -1$, $x > \frac{8}{5}$
38. $y = -x^2 + x$ or $y = -x(x - 1)$

Selected Answers

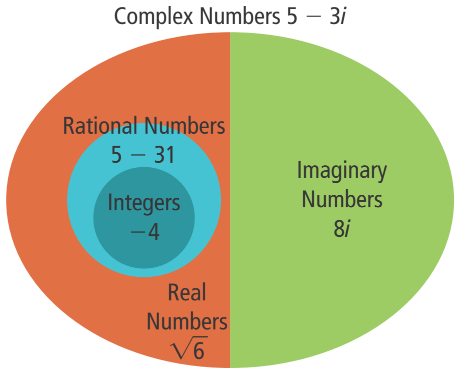
Topic 2

40. 39 ft by 27 ft; $x(x + 12) = 1053$; $x^2 + 12x = 1053$; $x^2 + 12x - 1053 = 0$; $(x + 39)(x - 27) = 0$; $x + 39 = 0$ or $x - 27 = 0$; $x = -39$ or $x = 27$; A dimension cannot be negative, so $x = 27$ ft is the width, and $x + 12 = 39$ ft is the length of each apartment. **42. a.** $(2x + 9)(2x + 14) = 9 \times 14 + 140$ or $4x^2 + 46x + 126 = 266$ **b.** 2.5 meters; $4x^2 + 46x + 126 = 266$; $4x^2 + 46x - 140 = 0$; $2(2x - 5)(x + 14) = 0$; $2x - 5 = 0$ or $x + 14 = 0$; $x = 2.5$ or $x = -14$; The walkway must have a positive width, so it is 2.5 meters wide. **44.** D

Lesson 2-4

2. The complex conjugate of $a + bi$ is $a - bi$. Their product is $a^2 + b^2$, a real number. **4.** The function will never cross the x-axis. The function's intersections with the x-axis represent solutions to the equation, because $y = 0$ anywhere along the x-axis. Since this equation has no real solutions ($3i$ and $-3i$ are both imaginary), the function will not cross the x-axis. **6.** $-6 + 12i$ **8.** $5i, -5i$ **10.** Tamara is not correct. Raising the number i to *even* integer powers will always result in 1 or -1. $i^2 = -1$, $i^4 = (i^2)^2 = (-1)^2 = 1$, $i^6 = -1$, $i^8 = 1$, and so on. But raising i to *odd* integer powers gives a different pattern: $i = \sqrt{-1}$, $i^3 = (i^2)(i) = -i$, $i^5 = i$, $i^7 = -i$, $i^9 = i$, and so on.

12.



14. $\pm i\sqrt{5}$ **16.** $\pm 3i\sqrt{2}$ **18.** $12 - 3i$ **20.** 11

22. $\frac{1 - 7i}{12}$ **24.** -33 **26.** 29 **28.** $-2 + i$

30. $6 + 6i$ **32.** $4 - 2i$ **34.** $(2x + 7i)(2x - 7i)$ **36.** $4(5a - 3i)(5a + 3i)$

38. $\left(\frac{1}{2}b - 5i\right)\left(\frac{1}{2}b + 5i\right)$ **40.** $9i, -9i$

42. $4i, -4i$ **44.** $i, -i$ **46. a.** $\frac{7}{10} - \frac{3}{5}i$

b. $\frac{4 + 2i + 1 - i}{(4 + 2i)(1 - i)} = \frac{5 + i}{6 - 2i} = \frac{5 + i}{6 - 2i}\left(\frac{6 + 2i}{6 + 2i}\right)$ $= \frac{28 + 16i}{40} = \frac{7}{10} + \frac{2}{5}i$ **c.** If you choose R_2 to be the complex conjugate of R_1, the total resistance will be a real number: $\frac{R_1 + R_2}{R_1 R_2} = \frac{4 + 2i + 4 - 2i}{(4 + 2i)(4 - 2i)} = \frac{8}{20}$ $= \frac{2}{5}$

48.

Number	*R, I, C*
$2 + i$	*C*
$5 - 0i$	*R*
$2i$	*I*
$(3 - i)^2$	*C*
$i^2 + 1$	*R*
$3i$	*I*
$(3 - i)(3 + i)$	*R*
$(3 + i) - (2 + i)$	*R*
$\sqrt{-14}$	*I*
$i(4 + i) - 3i$	*C*

Selected Answers

Topic 2

Lesson 2-5

2. No, if the leading coefficient is a number other than one, you can factor it out and then continue with completing the square. **4.** Use completing the square to write the quadratic in vertex form, $y = a(x - h)^2 + k$, in order to determine the vertex (h, k). **6.** $-2 + \sqrt{13}$, $-2 - \sqrt{13}$ **8.** $y = (x + 3)^2 - 15$; minimum $(-3, -15)$ **10.** 40 units **12.** The student did not add the value for completing the square, 64, to both sides of the equation. **14.** No; you cannot geometrically depict negative values or subtracted values. **16.** There is no x-term in this expression, so it would be difficult to use completing the square. Besides, this expression is easily factored, as it is the difference of two squares. **18.** 2, –4 **20.** –9, 1 **22.** $-2 \pm \sqrt{7}$ **24.** $(x - 9)^2 = 17$ **26.** $\left(x + \frac{3}{2}\right)^2 = 9$ **28.** $\left(x - \frac{3}{4}\right)^2 = \frac{149}{16}$ **30.** $-4 \pm 2i\sqrt{11}$ **32.** –6.5, 2.5 **34.** $1 \pm i\sqrt{\frac{10}{3}}$ **36.** $-3 \pm 2\sqrt{17}$ **38.** $1 \pm \sqrt{6}$ **40.** $-4 \pm \sqrt{\frac{29}{3}}$ **42.** 50.2 ft and 20.5 ft **44.** $y = (x - 7)^2 + 22$; minimum: (7, 22) **46.** $y = -3(x - 6)^2 + 15$; maximum: (6, 15) **48.** $y = 0.5(x + 0.5)^2 + 2$; minimum: (–0.5, 2) **50. a.** 160 months **b.** 11,440 owls **c.** 329.1 months **52.** –1.24, 5.24, (2, 21) **54. Part A** $x^2 + 24x + 140 = 280$; Let x represent the number of feet to add to the length and width. The new dimensions and the new area can be represented as $(14 + x)(10 + x) = 2(10)(14)$. Expand to determine the function. **Part B** 18.85 ft by 14.85 ft

Lesson 2-6

2. The discriminant, $b^2 - 4ac$, tells the number and type of solutions of a quadratic equation in standard form, $ax^2 + bx + c = 0$. **4.** Methods include factoring, completing the square, factor by grouping, and the Quadratic Formula. Calculators can help with graphs and estimating square roots. **6.** $-3 \pm \sqrt{19}$ **8.** $x = 4$; another way is to recognize and factor the perfect square trinomial. **10.** The student used b, not $-b$, in the numerator of the Quadratic Formula; the correct answer is $\frac{5}{2} \pm \frac{\sqrt{5}}{2}$. **12.** I first look to see if the quadratic expression has a special form like a perfect square or difference of squares. I next check to see if it is easily factored. If the coefficient of x is even, I consider completing the square. If I am not able to factor quickly, I will use the Quadratic Formula. **14.** The discriminant is $b^2 - 4ac = -19$, so there are no real solutions. There is no real number x such that $f(x) = 0$, so the graph will not cross the x-axis. The domain of the function is all real numbers, so must include $x = 0$; the graph intersects the y-axis at (0, 5).

Selected Answers

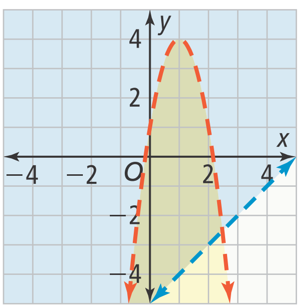
Topic 2

16. $x = 5$ **18.** $x = \frac{4}{5} \pm \frac{2}{5}i$ **20.** $x = 7$ or $x = -\frac{1}{3}$ **22.** 1 real solution **24.** 2 real solutions **26.** After 1 second **28.** $x = 1$ or $x = -\frac{11}{4}$ **30.** $x = 1$ or $x = -\frac{3}{2}$ **32.** $k = \pm 10$ **34.** $k < \frac{9}{16}$ **36. a.** 2 seconds **b.** after about 0.2 seconds and again at about 3.8 seconds **c.** about 4.2 seconds **d.** about 1.72 seconds **38.** B

Lesson 2-7

2. Dyani forgot to distribute the −1 when substituting for *y*. **4.** two solutions **6.** no solutions **8.** He did not check values for *b* that were less than 1. **10.** No; this can be seen graphically, and by the fact that the system reduces to one quadratic equation, which never has more than two solutions. **12.** no solutions **14.** Answers may vary. Sample: $m = 0$, $b = -3$ **16.** (6, 5) and (2,5) **18.** about 7.23 ft

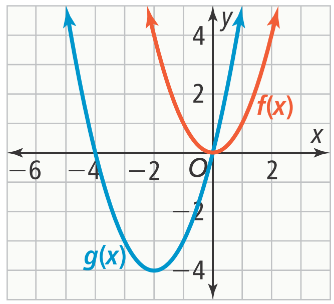
20.



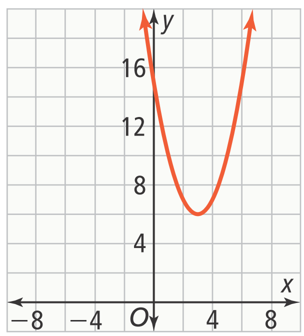
22. $x = 2$ **24.** The boulder will be 500 meters above the ground when it strikes the hillside, having traveled 1,500 meters horizontally. **26.** Yes; If you knew how far away the player was standing, you could be sure. **28.** D

Topic Review

2. Zero Product Property
4. discriminant **6.** standard form
8. The graph is to be translated 2 units left and 4 units down. It will open upward.



10. vertex (−3, 2), axis of symmetry $x = -3$, maximum $y = 2$, domain $(-\infty, \infty)$, range $(-\infty, 2)$ **12.** $f(x) = \frac{3}{4}(x-2)^2 + 1$ **14.** $g(x) = (x-2)^2 - 4$ **16.** vertex (3, 6), *y*-intercept (0, 15)

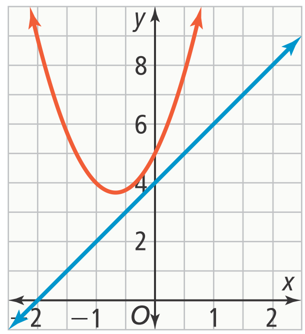


Selected Answers

Topic 2

18. $y = x^2 - 3x + 7$ **20.** Answers may vary. Sample: Calculate the vertex to find the maximum height of the ball. You can use the vertex to find the horizontal distance from the starting point and multiply by 2 to find the total distance. **22.** $x = -3$ or $x = 9$ **24.** $x = -1.5$ or $x = 0.5$ **26.** $x < -5$ or $x > 6$ **28.** $x \neq -6$ **30.** $13 - i$ **32.** $\frac{3}{2} - \frac{1}{2}i$ **34.** They did not change i^2 to -1; $(2 - 3i)(4 + i) = 11 - 10i$ **36.** $(x - 8)^2 = 28$ **38.** $x = 12 \pm \sqrt{226}$ **40.** $x = 2 \pm \frac{\sqrt{41}}{2}$ **42.** 5.04 seconds; The other root is negative, which is not an appropriate value for this situation.

44. $x = 8 \pm 2\sqrt{10}$ **46.** $x = \frac{9 \pm \sqrt{71}}{2}$ **48.** 2 real solutions **50.** $k = \pm 8$ **52.** below 25 kilometers per hour and above 79 kilometers per hour **54.** 0



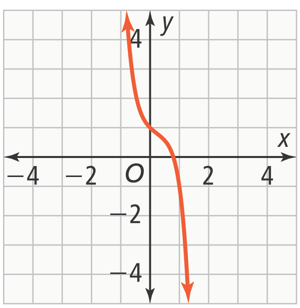
56. $(-1, 6)$, $(0, 7)$

Selected Answers

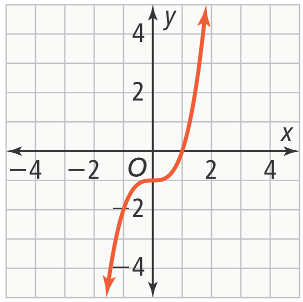
Topic 3

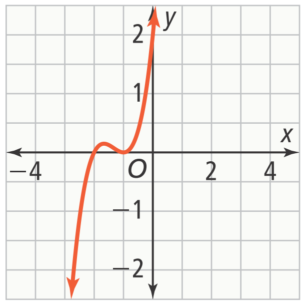
Lesson 3-1

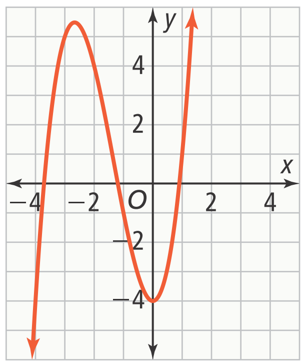
2. Allie did not write the polynomial function in standard form; instead, she said the exponent of the first term was the degree of the polynomial. The degree is 6. **4.** For a polynomial function with an even degree, as $x \to -\infty$ and $x \to +\infty$, y-values either both approach $+\infty$ or both approach $-\infty$. If the leading coefficient is positive, both ends approach $+\infty$. If negative, both ends approach $-\infty$. A polynomial function with an odd degree has end behavior for positive and negative x in opposite directions. If the leading coefficient is positive, as $x \to -\infty$, $y \to -\infty$, and as $x \to +\infty$, $y \to +\infty$. If the leading coefficient is negative, the end behavior is in the opposite direction. **6.** 4 **8.** As $x \to -\infty$, $y \to +\infty$. As $x \to +\infty$, $y \to +\infty$. **10.** $x = -4$, $x = -2$, $x = 1$, $x = 3$ **12.** approximately $(-0.5, 28)$ **14.** $f(x) = -2x^5 + 2x^4 - 2x^3 + x^2 - x + 1$; The polynomial function must have a negative leading coefficient and an odd degree because of the end behavior. The function must have a degree of at least 5 because there are 6 terms. The last term of the function is 1 because the y-intercept is (0, 1).



16. Check students' graphs. Sample:





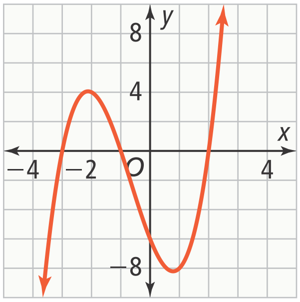


18. $f(x) = 2x^5 + x^4 + 5x^3 - 3x^2 + x - 6$; 5; 6; 2 **20.** $f(x) = -x^4 - x^3 + 5x^2 + 9x + 12$; 4; 5; -1 **22.** The leading coefficient, 7, is positive, so the graph opens upward. The degree is 4, which is even, so the end behaviors are the same. As x becomes infinitely positive or negative, the y-values approach $+\infty$.

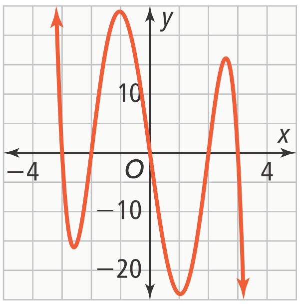
Selected Answers

Topic 3

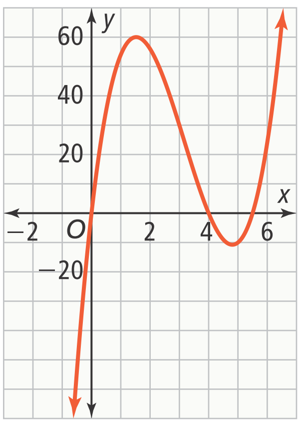
24. zeros: $x = -3$, $x = -1$, $x = 2$; turning points between -3 and -1, between -1 and 2



26.



28. a. $f(x) = 4x^3 - 38x^2 + 88x$
b. reasonable domain: $0 < x < 4$



c. x-intercepts: 0, 4, and 5.5; 0 and 4 represent the side lengths of the cut squares that will result in a box with 0 volume. The intercept at 5.5 is not meaningful, because it is not possible to cut two 5.5-inch corners from an 8-inch side. **d.** About 1.5 in. will create a box with a volume of about 60 in.^{3}.

30. 3, 7, −3, −3, 6 **32. Part A** The y-intercept for both functions is 8,000, which represents the population of the city in 2000. **Part B** For both functions, as $x \to \infty$, $y \to \infty$. The end behavior for $x \to -\infty$ is not relevant because the graphs of both functions begin at the y-intercept, (0, 8,000). **Part C** P's average rate of change, about 1,600, is greater than f's average rate of change, which is about 1,200.

Lesson 3-2

2. Chen did not distribute the factor of −1 over the terms $-mp$ and 1.
4. No, for example, $4 - 6$ is not a whole number. **6.** $2x^2y^2 - 5x^3 - x$
8. $x^3y^3 - 3x^2y^2 - 46xy + 48$ **10.** Use the algorithm for squaring a binomial or the FOIL method. **12.** $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ **14.** On the third line, the $-2y$ term should be positive, and the constant should be 14. The answer should be $3y^3 - 7y^2 - 5y + 14$.
16. The exponent of $3x^{-1}$ is not a whole number. **18.** $8x^3 + 2x^2 - 5x + 4$
20. $-5p^2q^2 + 2p^2q - 5pq^2 + 4pq$
22. $6c^3 - 23c^2 + 41c - 28$ **24.** No; x and 3 are both monomials, but $x + 3$ is not. **26.** The maximum value of f is 90; the maximum value of g is 32; end behavior of f: when $x \to -\infty$, $y \to -\infty$, and when $x \to \infty$, $y \to -\infty$; end behavior of g: when $x \to -\infty$, $y \to \infty$, and when $x \to \infty$, $y \to -\infty$ **28. a.** length $= 20 - x$; width $= 14 - x$; height $= \frac{1}{2}x$
b. $v(x) = (14 - x)(20 - x)\left(\frac{1}{2}x\right) = \frac{1}{2}x^3 - 17x^2 + 140x$ **c.** $D(x) = \frac{7}{2}x^3 - 51x^2 + 140x$ **30. a.** closed **b.** closed
c. closed **d.** open

Selected Answers

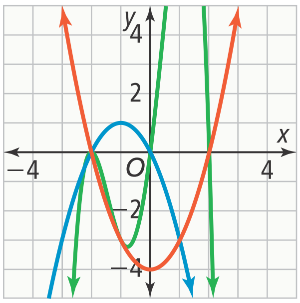
Topic 3

32. Part A $T(x) = -x^4 - 2x^3 + 4x^2 + 8x$

Part B

x	$P(x)$	$R(x)$	$T(x)$
−3	5	−3	−15
−2	0	0	0
−1	−3	1	−3
0	−4	0	0
1	−3	−3	9
2	0	−8	0
3	5	−15	−75

Part C



Part D The zeros of T include all the zeros of both P and R. **Part E** Because T is the product of R and S, T is positive when R and S are both positive or both negative.

Lesson 3-3

2. $(x + 5)^2$ is the same as $(x + 5)(x + 5)$. When expanding the polynomial, two of the terms are $5x$ and $5x$. The sum of these terms is $10x$. **4.** $8x^6$ and $27y^3$ are perfect cubes because $\sqrt[3]{8x^6} = 2x^2$ and $\sqrt[3]{27y^3} = 3y$. Use the difference of cubes to factor the polynomial. Substitute $2x^3$ for a and $3y$ for b: $(2x^2 - 3y)((2x^2)^2 + (2x^2)(3y) + (3y)^2) = (2x^2 - 3y)(4x^4 + 6x^2y + 9y^2)$.

6. Sample: Dakota did not multiply by the coefficient of the third term, which is 6. The third term of $(2g + 3h)^4$ is 216 g^2h^2. **8.** $x^2 + 6xy^3 + 9y^6$ **10.** $(2x^2 - y)(4x^4 + 2x^2y + y^2)$ **12.** $5xy^4$ **14.** $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ **16.** $d^4 - 4d^3 + 6d^2 - 4d + 1$ **18.** $27x^3 + 108x^2y + 144xy^2 + 64y^3$ **20.** Use the Binomial Theorem to write the expansion when $n = 4$: $C_0a^4 + C_1a^3b + C_2a^2b^2 + C_3ab^3 + C_4b^4$. Use Pascal's Triangle to write the coefficients: $a^4 + 4a^3b + 6a^2b^2 + 4ab^4 + b^4$. Identify a and b: $a = x$ and $b = i$. Substitute x for a and i for b: $(x)^4 + 4(x)^3(i) + 6(x)^2(i)^2 + 4(x)(i)^3 + (i)^4$. $i = \sqrt{-1}$, so $i^2 = -1$, $i^3 = -i$, and $i^4 = 1$. So, $(x - i)^4 = x^4 + 4x^3i - 6x^2 - 4xi + 1$. **22.** The coefficients in Pascal's Triangle when $n = 7$ are 1, 7, 21, 35, 35, 21, 7, and 1. There are 8 coefficients, so there will be 8 terms in the expansion of the expression $(-4y + z)^7$. **24.** $A = 5y^2$, $B = 5y^2$, $C = 25y^4$ **26.** Use the Difference of Squares Identity.$= (x^3)^2 - (y^3)^2$**28.** $x^2 - 81$ **30.** $9x^2 - 42x + 49$ **32.** $16x^4 - 36y^4$ **34.** $64 - x^4$ **36.** 396 **38.** 256 **40.** $(x^4 + 3)(x^4 - 3)$ **42.** $(2x + y^3)(4x^2 - 2xy^3 + y^6)$ **44.** $(2x + y^3)(2x - y^3)$ **46.** $(4x - 5y^2)(16x^2 + 20xy^2 + 25y^4)$ **48.** 945 **50.** 973 **52.** $x^3 + 9x^2 + 27x + 27$ **54.** $b^4 - 2b^3 + \frac{3}{2}b^2 - \frac{1}{2}b + \frac{1}{16}$ **56.** $8x^3 + 4x^2 + \frac{2}{3}x + \frac{1}{27}$ **58.** $d^4 - 12d^3 + 54d^2 - 108d + 81$ **60.** $n^5 + 25n^4 + 250n^3 + 1{,}250n^2 + 3{,}125n + 3125$ **62.** $256g^4 + 512g^3h + 384g^2h^2 + 128gh^3 + 16h^4$ **64. a.** $(s + 3)^3$ **b.** $s^3 + 9s^2 + 27s + 27$ **c.** $(s - 2)^3$ **d.** $s^3 - 6s^2 + 12s - 8$ **66.** 9, 40, 41 **68.** E

Selected Answers

Topic 3

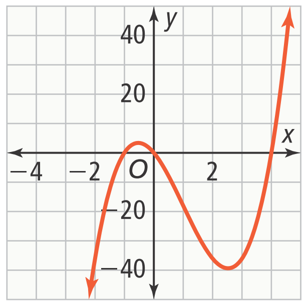
Lesson 3-4

2. No; Substitute -5 for x in the expression: $(-5)^3 + 2(-5)^2 - 4(-5) + 6 = -49$. So, the remainder is -49, not 149.
4. $x^2 - 4x + 4 + \frac{29x - 26}{x^2 + 8}$ **6.** $f(-2) = 2(-2)^4 + (-2)^2 - 10(-2) - 1 = 55$
8. $x^3 - 6x^2 + 12x - 5$ divided by $x - 1$; I multiplied $x^2 - 5x + 7$ by $x - 1$, and then added the remainder, 2. To check, use long division or synthetic division.
10. Alicia subtracted $-6x^2 - 6x$ from $-6x^2 + 6x$ and got a difference of 0. The difference is $12x$. So, the remainder should be $\frac{12x + 10}{x^2 + x}$. **12.** $(n, 0)$
14. $4x^2 - 6x + 9$; Multiply the answer by the divisor $2x + 3$ to check that the product is equal to $8x^3 + 27$; $(4x^2 - 6x + 9)(2x + 3) = 8x^3 + 12x^2 - 12x^2 - 18x + 18x + 27 = 8x^3 + 27$
16. $x^2 + 4x + 3$ **18.** $x^3 + 3x^2 + 6x + 18 + \frac{111}{2x - 6}$ **20.** $x^2 + x - 2$
22. $-x^3 + 4x^2 + 13x + 37 + \frac{99}{x - 3}$
24. 78 **26.** -36 **28.** The remainder is not 0, so $x - 3$ is not a factor of $P(x) = 8x^3 - 10x^2 + 28x - 16$.
30. The remainder is not 0, so $x - 2$ is not a factor of $P(x) = -x^5 + 12x^3 + 6x^2 - 23x + 1$. **32. a.** $x^2 + 5x + 4$ square units **b.** $x^2 - 3$ boxes **c.** $-5x - 2$ square units **34.** $3x^2 + 2x + 11$ mi/h **36.** A

Lesson 3-5

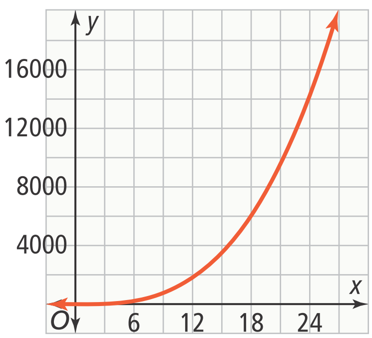
2. The student incorrectly applied the Zero Product Property. The zeros are 0, 5, and -2. **4.** The graph touches, but does not cross, the x-axis at $x = 2$, the exponent is an even number (multiple of 2).
6. You can use a graphing calculator to check the sketch. **8.** Graph them simultaneously on the graphing calculator to see if they coincide.
10. two real zeros at 2 and -2

12.



14. 0, 4; The graph crosses the x-axis at 0, and it touches the x-axis at 4.
16. $\pm\frac{2}{3}$, ±4; The graph crosses the x-axis at each zero. **18.** When $P(x)$ is positive, the company earns a profit. $P(x) > 0$ for $4 < x < 12$, so the company should make between 4,000 and 12,000 paddleboards. **20.** $x = -4$, $x = -3$, $x = 3$
22. $-3 < x < 0$ or $x > 3$ **24.** $x > -16$
26. Answers may vary. Sample: The baseball hits the ground a little past 2 seconds after being thrown. However, it is not being thrown from ground level, but from a height of 6.5 feet.
28. x – axis; $(x - 4)$

30. Part A



Selected Answers

Topic 3

Part B Venetta did not make a profit during the first two years. **Part C** The predicted amount of profit for 2020 is \$828,000.

Lesson 3-6

2. By the Conjugate Root Theorems, $-1 - 2i$ and $3 - \sqrt{5}$ are also roots. There are 4 known roots: $-1 + 2i$, $-1 - 2i$, $3 + \sqrt{5}$, and $3 - \sqrt{5}$. There could be other roots. So the polynomial function has a degree of 4 or greater. **4.** No; if the coefficients of the polynomial function are not real numbers, then $4 - 2i$ need not also be a root. **6.** possible roots: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$; rational roots: $-2, -1, 2, 3$ **8.** possible roots: $\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{2}{9}, \pm\frac{4}{1}, \pm\frac{4}{3}, \pm\frac{4}{9}, \pm\frac{8}{1}, \pm\frac{8}{3}, \pm\frac{8}{9}, \pm\frac{16}{1}, \pm\frac{16}{3}, \pm\frac{16}{9}$; rational roots: $-2, -\frac{2}{3}, \frac{2}{3}, 2$ **10.** $5 - 12i$ and $-9 + 7i$
12. $5 + 15i$ and $17 - \sqrt{23}$
14. $Q(x) = x^4 + x^3 + 4x^2 + 4x$
16. Answers may vary. Sample: $2x^3 + 7x^2 + x - 10$; Assume that the coefficients of the polynomial have no common integer factors other than 1 and -1. The denominators of the possible roots are 1 and 2. A number that has factors of 1 and 2 is 2. So, the leading coefficient must be 2 or -2. The numerators of the possible roots are 1, 2, 5, and 10. A number that has factors of 1, 2, 5, and 10 is 10. So the constant term must be 10 or -10. The leading term must have degree 3 since the equation is a third-degree polynomial. **18.** 2; Answers may vary. Sample: $P(x) = x^5 + 9x^3$; Because $3i$ is a root, $-3i$ is also a root. The factors are $(x + 3i)$ and $(x - 3i)$. Use the FOIL method to get $x^2 + 9$. In order for the equation to be a fifth-degree polynomial, multiply each term by x^3 to get $x^5 + 9x^3$. **20.** $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{4}{1}, \pm\frac{6}{1}, \pm\frac{12}{1}$ **22.** $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{4}, \pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{4}{4}, \pm\frac{8}{1}, \pm\frac{8}{2}, \pm\frac{8}{4}, \pm\frac{16}{1}, \pm\frac{16}{2}, \pm\frac{16}{4}$
24. 8 feet **26.** $\{-9, \sqrt{7}, -\sqrt{7}\}$ **28.** $\{-1, 1, 3i, -3i, 2, -2\}$ **30.** $P(x) = x^2 - 2x + 37$
32. $P(x) = x^3 - 3x^2 + 40x + 400$
34. length: $7 + \sqrt{5}$, or about 9.24 ft; width: $7 - \sqrt{5}$, or about 4.76 ft; height: 4 ft **36.** B, E **38. Part A** 7 real and 0 complex, 5 real and 2 complex, 3 real and 4 complex, 1 real and 6 complex **Part B** A polynomial equation with an odd degree has an odd number of real roots.

Lesson 3-7

2. An even function has the y-axis as a line of symmetry and is defined for all x, such that $f(x) = f(-x)$. An odd function has the origin as its point of symmetry and is defined for all x, such that $f(-x) = -f(x)$. **4.** neither
6. $V(x + 2) = x^3 + 4x^2 + 4x$ **8.** The graph should be reflected across the x-axis before being translated, because the coefficient, $-\frac{1}{2}$, is negative.
10. The parent function is even because it is given that it is quartic. Since the end behavior is opposite of the parent quartic function, it is a reflection over the x-axis. The point (2, 6) shows that the graph has shifted 2 units right and 6 units upward. **12.** Sample: $f(x) = 2x^4 + x^3 - 2$ has even degree but is not even. $f(x) = 2(-x)^4 + (-x)^3 - 2 = 2x^4 - x^3 - 2$, which is not equal to $f(x)$.

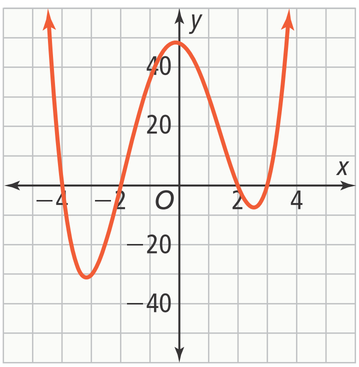
Selected Answers

Topic 3

14. neither **16.** even **18.** parent function: $y = x^4$; The leading coefficient of -1 reflects the graph across the x-axis; subtracting 8 shifts the graph down 8 units.
20. $f(x) = (x - 5)^4 + 10$ **22. a.** $g(x) = f(x) + 1{,}245 = 0.8x^3 + 6x^2 + 2{,}245$ **b.** a shift 1,245 units upward **c.** an increase of 1,245 weekly visitors
24. a. $V(x) = 55x^3 + 6x^2 - x$
b. $Z(x) = 220x^3 + 24x - 4x$ **26.** C

Topic Review

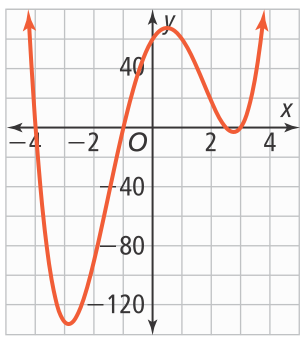
2. degree of the polynomial **4.** end behavior **6.** multiplicity of a zero
8. synthetic division **10.** zeros: $x = -4$, $x = -2$, $x = 2$, $x = 3$; turning points between -4 and -2, between -2 and 2, between 2 and 3



12. After 7 hours, Sadie's elevation is 18 meters below sea level. The x-intercepts are 1, 4, and 6. This means that after 1 hour, after 4 hours, and again after 6 hours, Sadie's elevation is 0 meters, or at sea level. The y-intercept is 24. After 0 hours, when Sadie starts climbing, her elevation is 24 meters above sea level. All of these values make sense. Sadie starts at an elevation that is 24 meters above sea level, then she descends below sea level, then ascends above sea level, and finally descends again below sea level.

14. $7y^4 + 2y^3 + 4y^2 - 5y + 5$
16. $25x^2 + 80x + 64$
18. $(x^2 - 4)(x^4 + 4x^2 + 16)$, or $(x + 2)(x - 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$ **20.** $x^4 - 8x^3 + 24x^2 - 32x + 16$ **22.** Polynomial coefficients are real numbers, multiplied by variables raised to whole-number exponents. After subtracting two polynomials, the coefficients of the difference are also real numbers. The exponents are still whole numbers. The difference is still a polynomial, and the set of polynomials is closed under subtraction. **24.** $x^3 - 8x + 13 - \frac{25}{x + 2}$
26. Find $f(2)$. $f(2) = (2)^3 + 8(2)^2 - 9(2) - 3 = 19$, so the remainder is correct.

28.



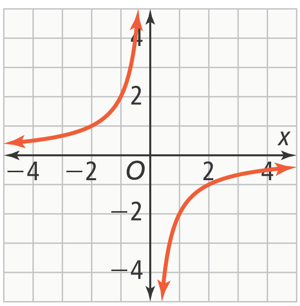
30. The graphs intersect at approximately $x = -2.7$, so the solution is approximately -2.7.
32. $\{-1, 5, 3i, -3i\}$ **34.** When the multiplicity of a zero is odd, the function crosses the x-axis. When the multiplicity of a zero is even, the graph touches the x-axis, but turns back and does not cross. **36.** even **38.** The graph of $f(x) = 0.5x^4 + 1$ is a vertical compression, not a vertical stretch, because the leading coefficient is less than 1.

Selected Answers

Topic 4

Lesson 4-1

2. As the time spent grilling increases, the amount of propane in the tank decreases. **4.** The product of x and y is not constant. (It is 32 in one column and 24 in the others.) **6.** vertical asymptote: $x = 5$; horizontal asymptote: $y = 3$; domain: the set of real numbers with $x \neq 5$; range: the set of real numbers with $y \neq 3$ **8.** When $k > 0$, the graph lies in Quadrants I and III; when $k < 0$, the graph lies in Quadrants II and IV. **10.** Division by zero is undefined. **12.** It is an inverse variation because as the number of persons sharing the rental fee increases, the cost per person decreases; $c = \frac{k}{p} = \frac{1500}{p}$. **14.** No; as the x-values increase, the y-values also increase. (This is a direct variation, where $k = 18$.) **16.** When $x = -1$, $y = -2$. **18.** vertical asymptote: $x = 0$; horizontal asymptote: $y = 0$; domain: the set of real numbers with $x \neq 0$; range: the set of real numbers with $y \neq 0$

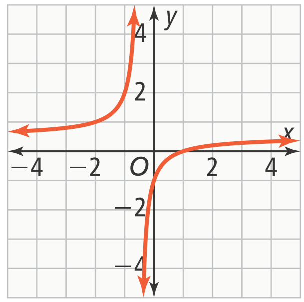


20. $t = \frac{4{,}800}{p}$; 24 min **22. a.** $V = \frac{2880}{R}$
b. When the resistance is 144 ohms, the voltage is 20 V. **24.** D

Lesson 4-2

2. They are expressed as fractions, or ratios. **4.** A rational function will have no vertical asymptotes when no value of x makes the denominator equal to 0; $f(x) = \frac{4x}{x^2 + 1}$

6. vertical asymptote: $x = -\frac{1}{2}$; horizontal asymptote: $y = \frac{1}{2}$

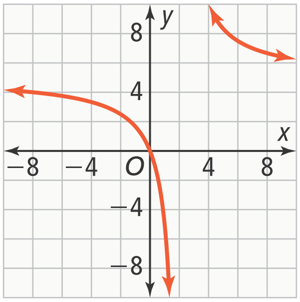


8. $y = \frac{a}{d}$; The rational function has a numerator and denominator of the same degree, so the horizontal asymptote is the ratio of the leading terms. **10.** At $x = -a$, there is a hole in the graph. When simplifying the function, the common factors cancel each other out and appear not to be part of the graph. But when substituting $-a$ for x in the original function, the denominator is equal to 0. Because dividing by 0 is undefined, there is a hole in the graph at $x = -a$. **12.** Substitute very large positive and negative values for x into the function. The y-values are approaching $\frac{1}{4}$, so the horizontal asymptote is $y = \frac{1}{4}$. As x approaches positive or negative infinity, the highest degree term greatly affects the end behavior of the graph and the other terms do not greatly affect the end behavior of the graph. Therefore, the ratio of the leading terms can be used to find the horizontal asymptote.

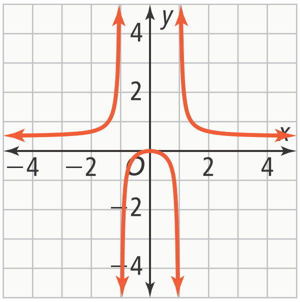
Selected Answers

Topic 4

14. $f(x) = 5 + \frac{10}{x-2}$; vertical asymptote: $x = 2$; horizontal asymptote: $y = 5$

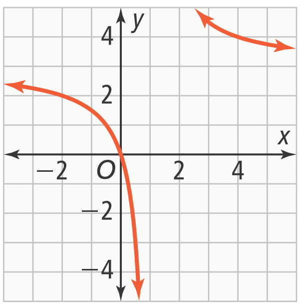


16. $f(x) = \frac{1}{2} + \frac{1}{2x^2 - 2}$; vertical asymptote: $x = -1$, $x = 1$; horizontal asymptote: $y = \frac{1}{2}$

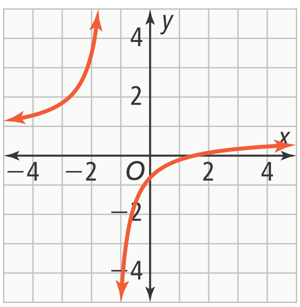


18. vertical asymptote: $x = 3$ and $x = 6$; horizontal asymptote: $y = 0$ **20.** vertical asymptote: $x = -1$ and $x = 1$; horizontal asymptote: $y = \frac{5}{2}$

22.



24.



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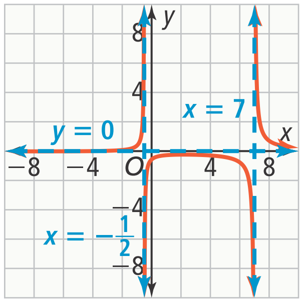

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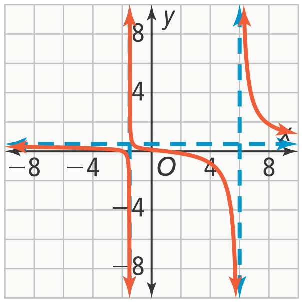
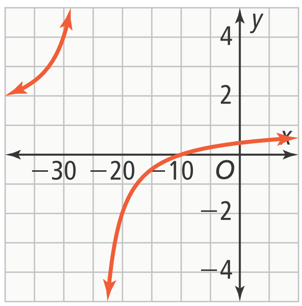


30. a. vertical asymptote: $x = -25$; horizontal asymptote: $y = 1$

b.



32. a. $0.1\overline{33}$ **b.** 2.85 hours **c.** **34.** B

Lesson 4-3

2. A rational expression is the ratio of two polynomials that has a domain of all real numbers except those for which the denominator equals zero; Sample: $\frac{3x^2 + 6x}{9x}$ **4.** When the denominator equals 0, fractions are undefined. **6.** $\frac{y+2}{y-3}$ for all y except -3, -2, and 3

Selected Answers

Topic 4

8. Sample: Any rational expression with the same numerator and denominator is equal to 1.

10. Sample: $\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \times \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{ad}{bc}}{\frac{cd}{dc}}$ $= \frac{\frac{ad}{bc}}{\frac{cd}{cd}} = \frac{\frac{ad}{bc}}{1} = \frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$

12. A rational number is the ratio of two integers, while a rational expression is a ratio of two polynomials. **14.** A rational expression is in simplest form when the numerator and denominator have no common factors other than 1. **16.** $-5, 0, 2$

18. $\frac{3x}{x-2}$ for all x except 2 and -5

20. $\frac{b(b+3)}{12(b+4)}$ for all real numbers except $a = 0$, $b = -4$ and 3

22. $\frac{(x+10)(x-1)}{(x-10)(x+1)}$, $x \neq -1, 0, 10$

24. $\frac{3(x-y)}{x+y}$ for all real numbers except $x = y$ or $x = -y$ **26.** $\frac{2x(3x+1)}{x-1}$ for all x except -1, 1, and 5 **28.** $\frac{(x-y)^3}{3(x+y)}$ for all x except $x = -y$ **30.** $\frac{x(x+6)}{(x+3)}$ $x \neq -6, -3, 0,$ $5, 6$ **32. a.** $\frac{r}{3}$ **b.** $\frac{3r}{8}$ **c.** cylinder B; $\frac{3}{8}r > \frac{1}{3}r$

34. The probability is $\frac{x}{6(x+1)}$. **36.** B

Lesson 4-4

2. A compound fraction is a fraction that has one or more fractions in the numerator and/or denominator. $\frac{\frac{1}{2}}{\frac{2x}{x+y}}$

4. The simplified sum or difference may not include some values of the domain that were eliminated during the simplification process.

6. $\frac{14}{x+1}$ **8.** $15x^3y^2$

10. $\frac{3y-4}{(y-2)(y-1)}$ **12.** The same basic procedures are followed for both. First, find a common denominator, then rewrite the expressions using the common denominator, perform the addition and/or subtraction, and then simplify. **14.** 1 **16.** The student did not factor either polynomial expression. The correct LCM is $3(3x+1)(x+2)$. **18.** $\frac{12y+5}{y^2+4y}$

20. $2y(y+6)(y+4)(y-4)$

22. $\frac{5y-1}{y(2y-1)}$ **24.** $\frac{2y-14}{15(y+5)}$ or $\frac{2(y-7)}{15(y+5)}$

26. $\frac{1}{x-1}$ **28.** $\frac{1}{a-b}$

30. a. $\frac{9x+7}{(x+3)(x-1)}$ **b.** 5 h

32. a. $\frac{180x+720}{x(x+8)}$ or $\frac{180(x+4)}{x(x+8)}$

b. ≈ 2.7 h **34.** C

Lesson 4-5

2. Sample: $\frac{2}{x+1} - \frac{1}{x-1} = \frac{-2}{x^2-1}$

4. Once the original equation has been multiplied by the LCD, the solution might yield values that are not solutions to the original equation.

6. $x = 3$ **8.** The time it takes working combined should be less than any of the individual times.

Selected Answers

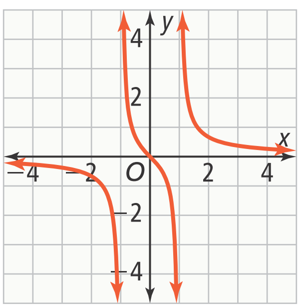
Topic 4

10. In the third step of her work, she incorrectly uses $x + 4$ rather than $x - 4$ on the right-hand side of the equation. This throws off every step thereafter; therefore; the solution is incorrect.
12. Oftentimes the steps involved in solving a rational equation transform it into a linear or quadratic equation.
14. Sample: The equation reduces to an identity; the solution is the set of all real numbers except −2. **16.** $x = 2$
18. $x = -2$ **20.** 32 hours
22. $x = -20$ **24.** $x = -2$ **26.** $1\frac{1}{5}$ h
28. $n = 2$ friends **30.** $h = 8$ hits **32.** D

Topic Review

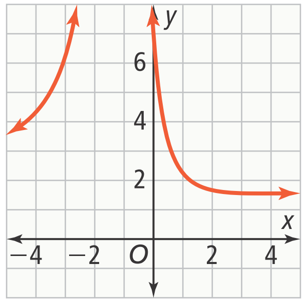
2. reciprocal function **4.** inverse variation **6.** asymptote
8. extraneous solution **10.** $\frac{1}{4}$
12. The parent reciprocal function is an inverse variation in which the constant of variation, k, is 1.
14. vertical asymptotes: $x = -7$, $x = -2$; horizontal asymptote: $y = 0$
16. vertical asymptote: none; horizontal asymptote: $y = \frac{1}{2}$

18.



vertical asymptote: $x = 1$, $x = -1$; horizontal asymptote: $y = 0$

20.



vertical asymptote: $x = -1$; horizontal asymptote: $y = 2$

22. day 3 **24.** $x^2 + 5x$; $x \neq -8, 0, 3$

26. $\frac{3}{x - 7}$; $x \neq -3, \frac{1}{2}, 7$ **28.** $\frac{2x^2 + x + 18}{(x + 6)(x - 1)}$

30. $\frac{x + 1}{x - 1}$ **32.** It is necessary to consider restrictions on variables when adding and subtracting rational expressions to avoid dividing by 0. **34.** $x = -1$
36. $x = \frac{3}{14}$ **38.** $x = -16$ or $x = 8$ (extraneous) **40.** Solve the rational equation. Then substitute the solution into the original equation to see if the solution makes the equation true. If the equation is not true, the solution is extraneous.

Selected Answers

Topic 5

Lesson 5-1

2. Kaitlyn made the exponent equal to the index. The index should be the denominator of the exponent, so the exponent should be $\frac{1}{3}$. **4.** $\left(75^{\frac{1}{5}}\right)^3$ can be simplified using the Power of a Power Property. Multiplying the exponents gives $75^{\frac{3}{5}}$. **6.** Yes, a rational exponent may be an improper fraction; $27^{\frac{4}{3}}$ is evaluated by finding the cube root of 27, which is 3, and raising it to the fourth power, which is 81. **8.** $\sqrt[3]{7^2}$
10. $p^{\frac{7}{4}}$ **12.** two **14.** $x \approx 5.95$ **16.** $-2xy^6$
18. Yes, Justice is correct; $(3x^3y)^5 = 3^5(x^3)^5y^5 = 243x^{15}y^5$;
20. $(x^4)^{\frac{1}{3}}$ is correct, but the denominator of the fraction, 3, is the index of the radical: $\sqrt[3]{x^4}$. **22.** 1; only one real number -8 times itself 3 times equals -512.
24. $4\sqrt{5}$ **26.** 7 **28.** ± 5 **30.** $729^{\frac{1}{6}}$
32. $(ab)^{\frac{1}{4}}$ **34.** $-3{,}125$ **36.** $\pm q^3 z$
38. $\pm vg^5$ **40.** ± 6 **42.** ± 2 **44.** 1 m
46. 0.025 or 2.5% **48.** yes; yes; no; no; no
50. Part A 36π ft^2 **Part B** 3 containers

Lesson 5-2

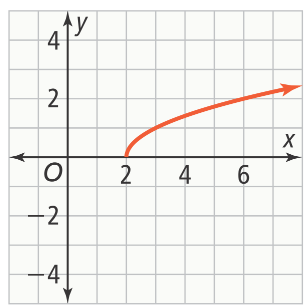
2. Check for perfect squares, cubes, etc., under the radical symbol, and make sure all radicals are removed from the denominator. **4.** The square root of a negative is an imaginary number, not a real number. **6.** $ab^6\sqrt[4]{a}$ **8.** $\frac{\sqrt[3]{12m}}{3m}$
10. $6\sqrt{5} + \sqrt{10}$ **12.** $\frac{3\sqrt{7} - \sqrt{35}}{4}$ **14.** k must be the product of a perfect square times 2.

16. Without simplifying first, you must estimate three separate square roots and then add those estimates. If they are simplified first, then they can be combined as $14\sqrt{3}$. Then only one square root would need to be estimated. **18.** Quotient of Powers Property; Simplify; Power of a Power Property; Write in radical form.
20. $18b\sqrt{b}\sqrt[3]{c^2}$ **22.** $\frac{4c^7}{9d^9}$ **24.** $4vw^3\sqrt[4]{v^3}$
26. $2x^2y^2\sqrt{2}$ **28.** $5f\sqrt[3]{f^2g^2}$ **30.** $\frac{\sqrt[3]{6n}}{2}$
32. $\frac{3\sqrt{3}}{a}$ **34.** $\frac{\sqrt[3]{3x^2y^2}}{3y}$ **36.** $\frac{\sqrt[4]{250x^3}}{5x}$
38. $10y\sqrt{5}$ **40.** $17\sqrt{2h}$
42. $16m^2 + 8m\sqrt{3} + 3$
44. $15 - 4\sqrt[3]{3}$ **46.** $\frac{60 - 20\sqrt{2}}{7}$
48. $\frac{-6x + 2x\sqrt{x}}{9 - x}$ **50.** 8.6% **52.** $20\sqrt{22}$ ft^2
54. D

Lesson 5-3

2. The graph of $g(x) = -\sqrt{x + 2} - 1$ is a translation 2 units left and 1 unit down from the parent function $f(x) = \sqrt{x}$, after a reflection over the x-axis. Parker didn't notice the negative symbol.

4.

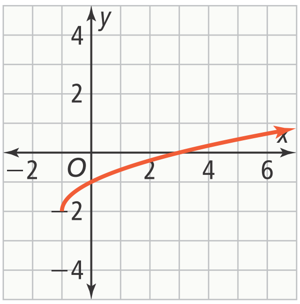


domain: $x \geq 2$; range: $y \geq 0$

Selected Answers

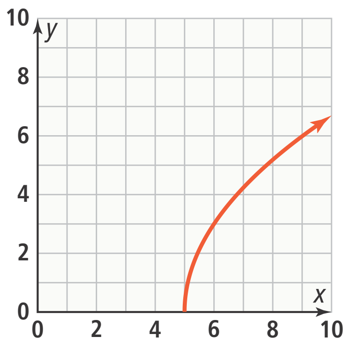
Topic 5

6.



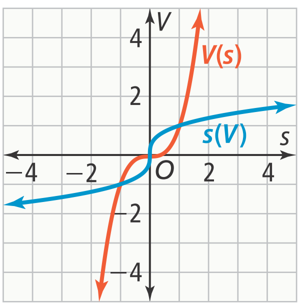
domain: $x \geq -1$; range: $y \geq -2$

8.



domain: $x \geq 5$; range: $y \geq 0$

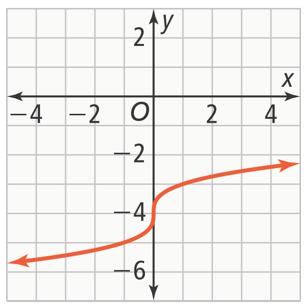
10.



a. $s(V) = \sqrt[3]{V}$ **b.** The graph of $s(V)$ is a reflection across the line $V = s$ of $V(s)$. This means the functions are inverses.

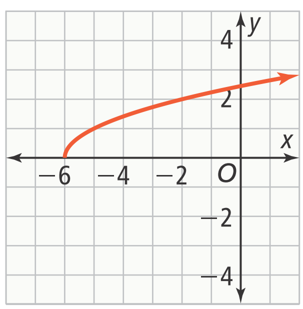
12. a. Answers may vary: $g(x) = \sqrt[3]{x+3} - 1$ **b.** Check students' graphs. **14.** Factor the radicand: $g(x) = \sqrt[3]{8(x+8)} - 3$. Use the Product Property of Radicals: $g(x) = \sqrt[3]{8}\,\sqrt[3]{8(x+8)} - 3$. Simplify: $h(x) = 2\sqrt[3]{x+8)} - 3$. The graph of $g(x)$ is the vertical stretch of the parent function by a factor of 2 and a translation of 8 units to the left and 3 units down. **16.** The domain changes with a horizontal translation of a radical function, unlike an absolute value function which continues to have all real numbers as the domain. The range can change with vertical translations of both functions.

18.



domain: all real numbers; range: all real numbers; The function is increasing.

20.



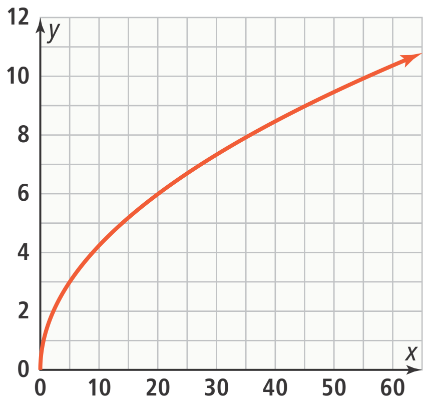
domain: $x \geq -6$; range: $y \geq 0$; The function is increasing.

Selected Answers

Topic 5

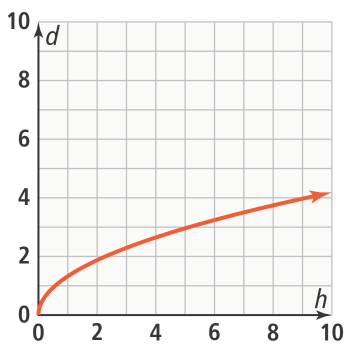
22. $f(x) = 4\sqrt{x}$; vertical stretch by a factor of 4 **24.** $f(x) = 3\sqrt{x - 5}$; vertical stretch by a factor of 3 and translation 5 units right **26.** $f(x) = \sqrt{x - 1} + 1$

28.



The minimum hull speed of the sailboat Chair wants to purchase is 6.7 knots and the maximum hull speed is 10.72 knots.

30. a.



b. 14.25 ft **32.** D

Lesson 5-4

2. Graph $y = \sqrt[3]{84x + 8}$ and $y = 8$. The graphs will intersect at $x = 6$. **4.** Neil didn't check for extraneous solutions. The real solution is 6; -3 is extraneous. **6.** $x = 27$ **8.** $x = -1$ **10.** $x = 6y^2 + 48$ **12.** $x = 10$ **14.** $x = -1$ **16.** $P = 4\sqrt{A}$; Sample: If $A = s^2$, then $s = \sqrt{A}$. Substituting $\sqrt{A}$ for s in the perimeter formula results in the given answer.

18. $x = 17$, $y = 7$ **20.** The process of solving an equation with two radicals is like solving an equation with one radical. Simplify the equation and square both sides to eliminate the radical; because there are two radicals, you have to square both sides a second time; Rewrite the equation in standard form, and then factor the equation using the Zero-Product Property to solve for the variable. There are additional steps needed to eliminate the second radical. **22.** 30.25 **24.** 81 **26.** $y = 338x^2$ **28.** $y = 81x^4$ **30.** $x = \frac{5}{4}$ and $x = 3$ **32.** $x = 7$ **34.** $x = -5$ and $x = 17$ **36.** $x = 8\frac{1}{16}$ **38.** $x = -\frac{3}{4}$ **40.** $d = \dfrac{s^2}{252.81f}$ **42.** 13 ft

44.

y	x	z
0	462	804
–3	145	439
–10	1.25	5.5
3	1,679.5	16

46. Part A $M = \dfrac{v^2 r}{2G}$
Part B $M \approx 5.99 \times 10^{24}$

Lesson 5-5

2. A composite function is a method of combining functions such that the output of one function is the domain of the other function. Example: Given $f(x) = 2x$ and $g(x) = x + 1$, the composite function $f \circ g = f(g(x)) = 2(x + 1) = 2x + 2$. **4.** The operations of subtraction and division are not commutative. **6.** $3x^2 + 3x + 2$ **8.** $x^3 - 2x^2 - 7x - 4$ **10.** $\dfrac{x - 4}{x^2 + 2x + 1}$ or $\dfrac{x - 4}{(x + 1)^2}$ for all real numbers x, when $x \neq -1$

Selected Answers

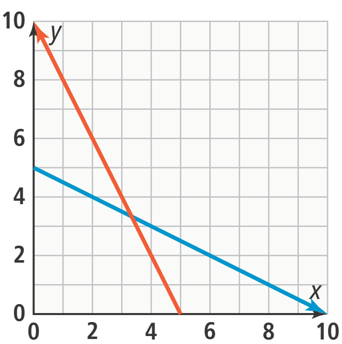
Topic 5

12. In general $f \circ g$ and $g \circ f$ are usually not equal. Sample: If $f(x) = x + 1$ and $g(x) = 4x$, then $(f \circ g)(x) = 4x + 1$ and $(g \circ f)(x) = 4x + 4$. **14.** When $2x + 1$ was substituted into $3x^2 - x + 2$ for x and when $-x$ was replaced with $2x + 1$, the negative sign was not distributed. **16.** $(f \circ g)(x) = \{(5, 1), (3, 7), (1, 2), (2, 8)\}$ **18.** If $f(x) = \frac{1}{2}x$ and $g(x) = 2x$, then $f(g(x)) = x$.
20. $f + g = 2x^2 + 8x + 1$
22. a. $r(x) = 600x - 4x^2$ **b.** \$17,600
24. -89 **26.** -21 **28.** $4x^2 - 38x + 90$
30. It is better for Kayden if the 10% discount is applied before the \$50 instant rebate because he will save \$5 more.
32. a. \$72.25 **b.** \$71.50 **c.** Since \$71.50 < \$72.25, it is a better deal for the consumer if the 15% discount is applied prior to the \$5 off discount. **34** A, B, E
36. Part A $K = \frac{5}{9}(F - 32) + 273$
Part B 270 K

Lesson 5-6

2. No; the inverse is $f^{-1}(x) = \frac{x - 1}{3}$.
4. $f^{-1}(x) = -2x + 10$

6.



8. Yes; each independent value is paired with only one dependent value.
10. The student took the square root of both sides before adding 4 to both sides. The inverse is $f^{-1}(x) = \sqrt{x + 4}$.

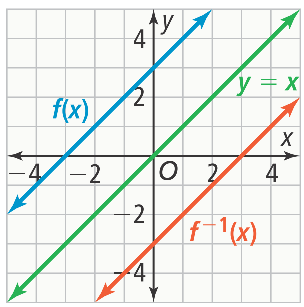
12. $y \neq 3$, 4, or 6 because after the x- and y-coordinates are switched, if the value of y is 3, 4, or 6, each independent value would not be paired with only one dependent value.
14. The relation is not a function because each independent value is not paired with only one dependent value. The inverse of the relation is a function because each independent value is paired with only one dependent value.

16.

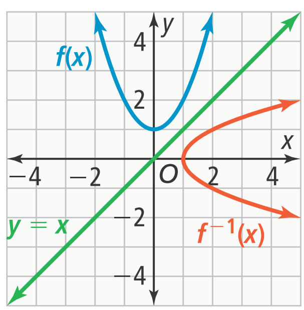
x	9	3	-4	8	-6	3
y	-2	-1	0	1	2	3

The inverse is not a function.

18. $f^{-1}(x) = x - 3$; The inverse of f is a function.



20. $f^{-1}(x) = \pm\sqrt{x - 1}$; The inverse of f is not a function.



Selected Answers

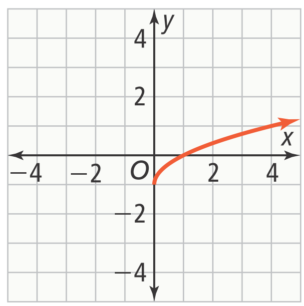
Topic 5

22. $f(x) = x^2 + 4x + 4 = (x + 2)^2$, so the graph of the parabola has its vertex at $(-2, 0)$. Restrict the domain of f to $x \geq -2$. Then $f^{-1}(x) = \sqrt{x} - 2$, $x \geq 0$.
24. $f(x) = x^2 - 2$, so the graph of the parabola has vertex $(0, -2)$. Restrict the domain of f to $x \geq 0$. Then $f^{-1}(x) = \sqrt{x + 2}$, $x \geq -2$.
26. $f^{-1}(x) = \pm\sqrt{x + \frac{5}{2}}$; domain is $x \geq -5$
28. $f^{-1}(x) = \frac{x - 10}{3}$ for all real values of x.
30. $f(g(x)) = 2(\frac{1}{2}x + 9) - 9 =$
$x + 18 - 9 = x + 9$;
$g(f(x)) = \frac{1}{2}(2x - 9) + 9 =$
$x - \frac{9}{2} + 9 = x + \frac{9}{2}$, so f and g are not inverses.
32. $r(V) = \sqrt{\frac{3V}{\pi h}}$; r is about 7.6 cm
34. a. $C = 75n + 100$ **b.** $n = \frac{C - 100}{75}$; The inverse represents the number of hours the DJ is hired in terms of the cost. **c.** 6 h **36. a.** yes **b.** no **c.** yes **d.** yes **e.** no **38. Part A** $k^{-1}(x) = \pm\sqrt[4]{x}$; $m^{-1}(x) = \sqrt[5]{x}$; $n^{-1}(x) = \pm\sqrt[6]{x}$ **Part B** $k^{-1}(x)$ is not a function; $m^{-1}(x)$ is a function; $n^{-1}(x)$ is not a function. **Part C** Answers may vary. Sample: Functions that have an odd power have inverses that are functions. Functions that have an even power have inverses that are not functions.

Topic Review

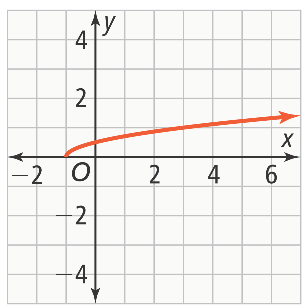
2. index **4.** like radicands **6.** nth root **8.** reduced radical form **10.** 4
12. $3x^4$ **14.** $y = 5$ **16.** The numerator of a rational exponent is the power of the radicand. The denominator of a rational exponent is the index of the radical. The nth root of a number x is expressed as $x^{\frac{1}{n}} = \sqrt[n]{x}$. **18.** x^7y^5
20. $5x^3$ **22.** $n + 2\sqrt{7n} - 21$ **24.** 39
26. $-6 + 6\sqrt{2}$ **28.** $-\frac{2 + \sqrt{6}}{3}$ **30.** The expressions were incorrectly subtracted as like expressions; $5\sqrt{18} - \sqrt{27} = 15\sqrt{2} - 3\sqrt{3}$.

32.



The domain of the function is $x \geq 0$. The range of the function is $y \geq -1$. The function is increasing over the entire domain.

34.

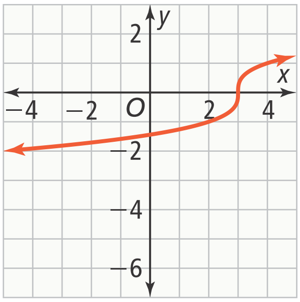


The domain of the function is $x \geq -1$. The range of the function is $y \geq 0$. The function is increasing over the entire domain.

Selected Answers

Topic 5

36.



The domain of the function is all real numbers. The range of the function is all real numbers. The function is increasing over the entire domain.

38. Factor the expression under the radical: $g(x) = \sqrt[3]{8(x-3)} + 1$. Use the Product Property of Radicals: $g(x) = \sqrt[3]{8} \bullet \sqrt[3]{x-3} + 1$. Simplify: $g(x) = 2\sqrt[3]{x-3} + 1$. $g(x)$ has a vertical stretch by a factor of 2, a horizontal shift to the right 3 units, and a vertical shift up 1 unit of $f(x)$.

40. $x = 729$ **42.** $x = -9$
44. $x = 0$ (extraneous), $x = 7$
46. Sample: $\sqrt{x-5} = -2$; Square both sides: $x - 5 = 4$. Add 5 to both sides: $x = 9$. Substitute 9 for x in the original equation: $\sqrt{9-5} = -2$. Simplify: $2 \neq -2$. So, there are no real solutions. **48.** $4x + 6$ **50.** 20
52. all real numbers; $x \neq 0$; $x \neq 6$
54. $f^{-1}(x) = \pm\sqrt{\frac{-x+3}{4}}$ **56.** $f^{-1}(x) = \frac{x-5}{9}$
58. No; Jamie added 9 to both sides and then squared both sides instead of squaring both sides and then adding 9 to both sides. The inverse is $f^{-1}(x) = x^2 + 9$.

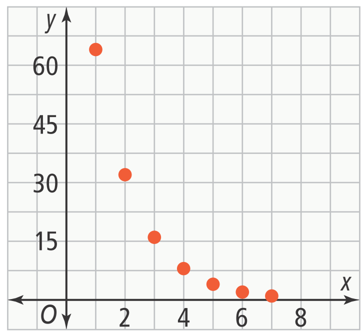
Selected Answers

Topic 6

Lesson 6-1

2. In exponential functions, constants are raised to powers that are variables. In polynomial and rational functions, variables are raised to powers that are constants. **4.** Sample: Growth and decay functions have the same domain, range, and asymptotes but different end behaviors. **6.** decreasing; $1 - 0.6$ **8.** The truck modeled by $f(x)$ will be worth more after 5 years; $f(5) = 15.53$ and $g(5) = 10.92$
10. $f(x) = 8{,}000(1.015)^x$; \$8,878.76
12. 346,904; This is the population for the first known year of data. The year 2000 acts as 0. **14.** domain: all real numbers; range: $\{y \mid y > 0\}$; y-intercept: 5; asymptote: x-axis; end behavior:
As $x \to -\infty$, $y \to 0$. As $x \to \infty$, $y \to \infty$.
16. domain: all real numbers; range: $\{y \mid y > 0\}$; y-intercept: 4; asymptote: x-axis; end behavior: As $x \to -\infty$, $y \to \infty$. As $x \to \infty$, $y \to 0$. **18.** growth; $1 + 1.5$; 1.5; 150% **20.** decay;
$1 - \frac{7}{10}$; $\frac{7}{10}$; 70% **22.** 10.8; 17.8
24. $f(x) = 4{,}007(0.9964)^x$; 3,728
26. a. decay; The base is less than 1. **b.** the initial number of teams in the tournament **c.** 50%; the rate of decay is $\frac{1}{2}$

d.



domain: {1, 2, 3, 4, 5, 6, 7}; range: {1, 2, 4, 8, 16, 32, 64}; the function only makes sense for 7 rounds. After the seventh round, there is only one team remaining.

28. a. yes **b.** no **c.** no **d.** yes **e.** no **f.** no

30. Part A $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{21.5}}$
Part B about 143 hours

Lesson 6-2

2. Yori confused the annual interest rate and the monthly rate.
4. a represents the initial value, b represents the decay factor, x represents the amount of time that has passed, and y represents the value at a given time. **6.** 0.45%; 1.3%
8. \$1,641.26 **10.** $y = 0.071(2.08)^x$
12. The student compounded the interest monthly instead of quarterly. A = \$7,935.16.

Selected Answers

Topic 6

14. $y = 2{,}954(1.08)^x$; $y = 3{,}001(1.078)^x$; To find the exponential model using data points, find the growth rate between two consecutive data points and that growth rate and one of the points to find the initial value. To find the model using a calculator, enter the data and find the exponential regression. \$9,259 **16.** 1,367.31 **18.** 4,469.79 **20.** 1,150.27 **22.** $y = 0.757(1.786)^x$ **24.** $y = 0.510(1.558)^x$ **26.** after about 28 minutes **28.** $y = 80{,}000(1.056)^t$; 0.0149%; Multiply the exponent by $\frac{365}{365}$ so that the model compounds daily, then calculate $(1.056)^{\frac{1}{365}}$ to find the daily rate. **30.** $y = 20.361(0.743)^x$; 1.221×10^{-22}g **32.** C

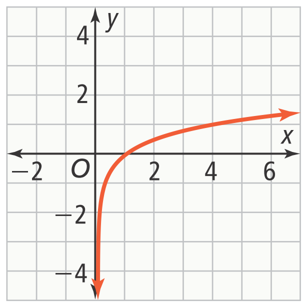
Lesson 6-3

2. Amir possibly brought the negative sign outside the expression, which is not an equivalent step. **4.** You can convert the equation to logarithmic form, $t = \log 656$, which also solves the equation. **6.** $\ln 54.6 \approx 4$ **8.** $e^{3.22} \approx 25$ **10.** -2 **12.** $\ln\left(\frac{7}{4}\right)$, or 0.5596 **14.** The student did not convert to logarithmic form correctly. The solution should be $t = \ln 6.125$. **16.** $\{x \mid 0 < x < 1\}$ **18.** For the natural logarithm the base is e not 10. **20.** $\log y = x$ **22.** $\log_a y = x$ **24.** $\ln 0.0498 \approx -3$ **26.** $\log_7 343 = 3$ **28.** $8^2 = 64$ **30.** $2^{-5} = \frac{1}{32}$ **32.** undefined **34.** 5 **36.** -2 **38.** a **40.** -0.6198 **42.** 3.7257 **44.** 1.0986 **46.** 2.6417 **48.** 0.6990 **50.** 3.1 **52.** Peter's account **54. a.** 183°F **b.** 19 minutes **56.** C

Lesson 6-4

2. This is the range of the function. The domain is $\{x \mid x > 0\}$.

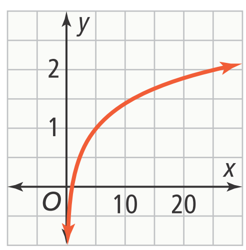
4.



domain: $\{x \mid x > 0\}$; range: all real numbers; intercept: x-intercept 1; asymptote: y-axis; end behavior: As $x \to 0$, $y \to -\infty$. As $x \to \infty$, $y \to \infty$.

6. $x = e^{\frac{y}{5}} - 1$; This function gives x, the number of minutes after the release of a song, in terms of the number of downloads in hundreds, y. **8.** $g(x)$ is shifted 7 units to the right. **10.** $w = \log_{1.6}(m - 2)$; the number of weeks after opening when the number of members reaches m

12.

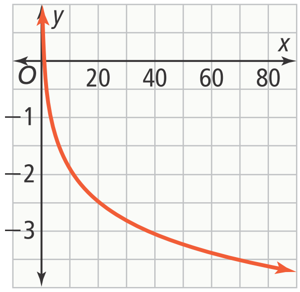


domain: $\{x \mid x > 0\}$; range: all real numbers; x-intercept: 1; asymptote: y-axis; end behavior: As $x \to 0$, $y \to -\infty$. As $x \to \infty$, $y \to \infty$.

Selected Answers

Topic 6

14.



domain: $\{x \mid x > 0\}$; range: all real numbers; x-intercept: 1; asymptote: y-axis; end behavior:
As $x \to 0$, $y \to \infty$. As $x \to \infty$, $y \to -\infty$.
16. vertical shrink of $\frac{1}{2}$; asymptote: y-axis (same as parent function); x-intercept: 1 (same as parent function) **18.** Reflection of $f(x)$ shifted 0.5 units to the left.
20. $f^{-1}(x) = \log_{\frac{1}{2}} x + 1$ **22.** $f^{-1}(x) = \frac{2^x}{8}$
24. $f^{-1}(x) = 2^{\frac{(x-2)}{4}} + 3$ **26.** 0.07; 0.05
28. a. magnitude 9.2 **b.** The size of the arc can add between 2.2 and 3.7 to the surface wave magnitude, meaning the arc size can affect the surface wave magnitude by up to 1.5.

Lesson 6-5

2. This makes it easier to find the value of the expression on the calculator.
4. $2\log_6 7 - \log_6 5$ **6.** $10^{-8.9}$ or 1.259×10^{-9} **8.** To expand, separate the factors inside the logarithm into separate sums of logarithms. To write a single logarithmic expression, combine sums of logarithms into a single logarithm of the product.
10. Emma can check if $6^{2.387} = 72$.
12. The student applied the change of base property incorrectly.
14. $\log_5 2 - \log_5 3$ **16.** $\ln 2 + 5 \ln x$
18. $\ln \left(\frac{x^9}{y^6}\right)$ **20.** $\log (8100x^4)$
22. $\log_3 (256c^5d^7)$ **24.** 1.585 **26.** 1.099
28. 1.131 **30.** $\left(\frac{\log 4}{\log 3}\right)$; 1.262
32. $\left(\frac{\log 10}{\log 8}\right)$; 1.107 **34.** $\left(\frac{\log 100}{\log 7}\right)$; 2.367
36. a. 80 dB **b.** 10^{-8} W/m^2 **c.** 1,000 times **38.** B, C, A, D **40. Part A** 2.3 **Part B** $10^{0.9}$, or about 7.94, times greater **Part C** 150 times

Lesson 6-6

2. Jordan is incorrect. The equation contains a variable in the base but not in the exponent. **4.** 2 **6.** −4 (extraneous); 11 **8.** −3.82 **10.** common log; You could use either, but it is easier to use the common log because of the 10 on the left side. **12.** addition, product property of logarithms, property of equality for logarithmic equations, expansion, subtraction, factoring; Both solutions are correct, but −1 is extraneous. **14.** This property allows you to rewrite the exponents as products, enabling you to solve the equation. **16.** 1 **18.** −1.5; 3 **20.** 6
22. 1.4406 **24.** −0.5638 **26.** 4.0643
28. 0.09 **30.** −0.1111 (extraneous); 2
32. 0.25 (extraneous)
34. 0; −1 (extraneous) **36.** 1.077
38. $x \approx 0.604$ and $x \approx 7.012$
40. $x \approx 0.602$ **42.** 0.068 **44.** A, D
46. Part A $f(t) = 94.62 - 15 \log (t + 1.1)$
Part B about 8 years

Selected Answers

Topic 6

Lesson 6-7

2. Denzel thought the common ratio was 7, but 0(7) does not equal 7, so the sequence is not geometric.
4. The terms increase; the terms decrease. **6.** $\frac{1}{4}$; $-\frac{1}{4}$, $-\frac{1}{16}$, $-\frac{1}{64}$

8. –5; 1,250, –6,250, 31,250

10. 160 points; 310 points
12. He added terms 5 and 6 together. The actual answer is 432. **14.** 33 feet

16. yes; $a_n = \begin{cases} 1, \text{ if } n = 1 \\ -3a^{n-1}, \text{ if } n > 1 \end{cases}$

18. no **20.** no

22. $a_n = \begin{cases} 1{,}024, \text{ if } n = 1 \\ \frac{1}{2}a_{n-1}, \text{ if } n > 1 \end{cases}$

24. $a_n = \begin{cases} 35, & n = 1 \\ 2a_{n-1}, & n > 1 \end{cases}$

26. $a_n = \left(\frac{2}{3}\right)^{n-1}$

28. 252 **30.** –4,372

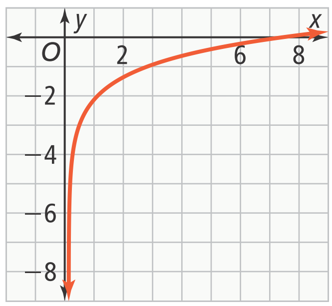
32. $\sum_{n=1}^{8} 8(2)^{n-1}$; 2,040

34. $\sum_{n=1}^{5} \frac{1}{5}\left(\frac{1}{2}\right)^{n-1}$; $\frac{31}{80}$

36. 7 **38.** No, she will only have $1,928.91 saved in the account.
40. $9,211.88 **42.** D

Topic Review

2. natural logarithm **4.** decay factor
6. common logarithm **8.** domain: all real numbers; range: $y > 0$; y-intercept: 400; asymptote: $y = 0$; end behavior: As $x \to -\infty$, $y \to \infty$. As $x \to \infty$, $y \to 0$.
10. $1,558.17 **12.** domain: all real numbers; range: $y > 0$; y-intercept 0.5; asymptote: $y = 0$ **14.** $6,057.19
16. $f(x) = 35.12(1.18)^x$ **18.** $16,559.53
20. log 100 = 2 **22.** $e^x = 20$ **24.** 4
26. –0.598 **28.** 49 **30.** domain: $x > 0$; range: all real numbers; x-intercept: 7.389; asymptote: $x = 0$; as $x \to 0$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$



32. $f^{-1}(x) = \log_5 8x + 2$

34. ln 4 **36.** 2.183 **38.** $\frac{\log 486}{\log 7}$; 3.179

40. 0 **42.** 8.220 **44.** –4; 8 **46.** 0.069

48. yes **50.** $a_n = \begin{cases} -2, \text{ if } n = 1 \\ 5a_{n-1}, \text{ if } n > 1 \end{cases}$

52. $121\frac{40}{81}$ **54.** 17,190 years

Selected Answers

Topic 7

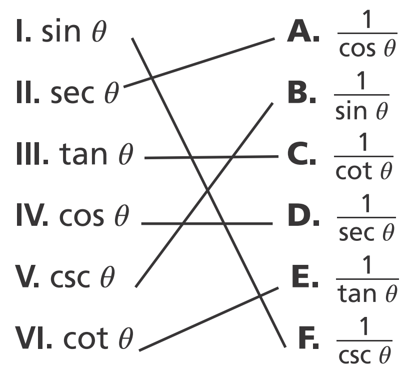
Lesson 7-1

2. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$; It is the reciprocal of $\frac{\text{hypotenuse}}{\text{adjacent}}$, which is sec θ. Cosine and sine are related through cofunction identities, not reciprocal identities.
4. All right triangles have one 90° angle, so the other two angles are always complementary. One angle's adjacent side is the other angle's opposite side, so the cosine of one angle must equal the sine of the other $(90° - \theta)$. **6.** Both ratios have the length of the hypotenuse as their numerator. Since this length has to be greater than either of the lengths of the opposite or adjacent sides, the ratio will be greater than 1 or less than −1. **8.** $\sin\theta = \frac{5}{13}$
10. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$ **12.** $\sec\theta = \frac{1}{\cos\theta}$
14. $\sec\theta = \csc(90° - \theta)$ **16.** 7.3 feet
18. Sine is opposite over hypotenuse. The hypotenuse will always be longer than the other sides, and none of the sides can be negative, so sine is $\frac{\text{opp}}{\text{hyp}}$, which is always between 0 and 1.

20. $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\text{adjacent}}{\text{hypotenuse}}} = \frac{\text{hypotenuse}}{\text{adjacent}}$

22. 4 ft **24.** $\sin\theta = \frac{8}{10}$, $\cos\theta = \frac{6}{10}$, $\tan\theta = \frac{8}{6}$, $\csc\theta = \frac{10}{8}$, $\sec\theta = \frac{10}{6}$, $\cot\theta = \frac{6}{8}$
26. $\sin\theta = \frac{16}{20}$, $\cos\theta = \frac{12}{20}$, $\tan\theta = \frac{16}{12}$, $\csc\theta = \frac{20}{16}$, $\sec\theta = \frac{20}{12}$ **28.** $\sin\theta = \frac{48}{52}$, $\cos\theta = \frac{20}{52}$, $\tan\theta = \frac{48}{20}$, $\csc\theta = \frac{52}{48}$, $\cot\theta = \frac{20}{48}$ **30.** $\sin\theta = \frac{\sqrt{2}}{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$
32. $\sec\theta = \csc(90° - \theta)$ **34.** 24.5 ft
36. 41.4 in.

38.

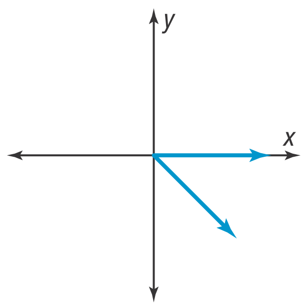


40. Part A about 12 ft **Part B** about 32 ft

Lesson 7-2

2. θ and its reference angle can be supplementary, but they do not have to be. If θ is in Quadrant I, for example, the reference angle is equal to θ.
4. Since the circumference of a unit circle is 2π, each quadrant covers $\frac{1}{4}$ or $\frac{\pi}{2}$. You can compare the angle measures in radians to 0, $\frac{\pi}{2}$, π, and $\frac{3\pi}{2}$ to determine in which quadrant the angle falls.
6. 65° **8.** Quadrant I **10.** −90° **12.** 60°
14. $-\frac{\pi}{6}$ or $\frac{11\pi}{6}$ **16.** Quadrant II
18. The student used the formula to solve for radians, not degrees: $\frac{\pi}{2} \times \frac{180}{\pi} = \frac{180\pi}{2\pi} = \frac{180}{2} = 90°$.
20. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **22.** 240°, −120°, 600°
24. 285° **26.** 56°

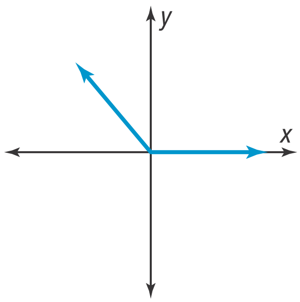
28.



Selected Answers

Topic 7

30.



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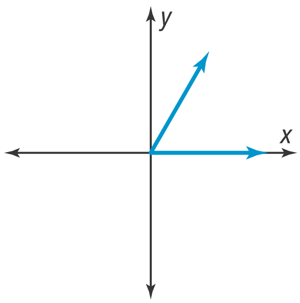
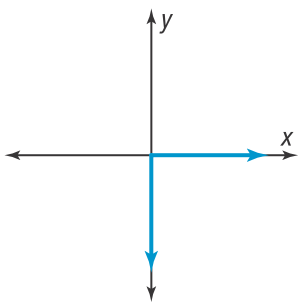


34.



36. 0.35 or $\frac{\pi}{9}$ **38.** 150° **40.** about 3.38 km **42.** ≈ 0.31 radians **44.** B

Lesson 7-3

2. Hugo may have thought that an angle in standard position, with measure $\frac{5\pi}{2}$, is in Quadrant III or IV, where the sine is negative. **4.** They have the same reference angle and are both in quadrants (I and IV respectively) where cosine is positive.

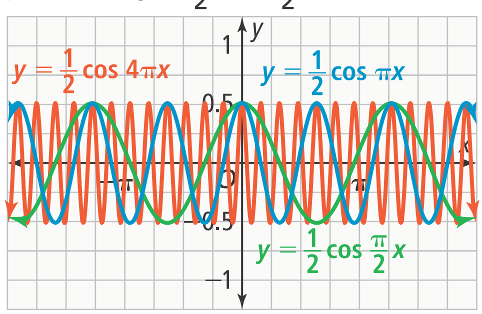
6. $\sin 120° = \frac{\sqrt{3}}{2}$; $\cos 120° = -\frac{1}{2}$

8. $\cos\theta = -\frac{\sqrt{3}}{2}$

10. $\tan(-45°) = -1$ **12.** No, the tangent of 270° is undefined because $\cos 270° = 0$.
14. The student found the cosine of 135° rather than its reciprocal function, secant. The secant of 135° is $\sqrt{2}$.
16. No, a reference angle must be an acute angle, which by definition must measure between 0° and 90°.
18. $\frac{5}{3}\pi$ radians
20. $\sin\frac{5\pi}{6} = \frac{1}{2}$; $\cos = \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$
22. $\sin 270° = -1$; $\cos 270° = 0$; coordinates of terminal point are $(0, -1)$. **24.** $\sin\theta = \frac{15}{17}$ **26.** $\tan\frac{7\pi}{3} = \sqrt{3}$
28. $\sec -315° = \sqrt{2}$; $\csc -315° = \sqrt{2}$; $\cot -315° = 1$ **30.** $\sec 750° = \frac{2\sqrt{3}}{3}$; $\csc 750° = 2$; $\cot 750° = \sqrt{3}$ **32.** Sample: The point $(4\sqrt{2}, 4\sqrt{2})$ represents a position that is about 5.7 km east and about 5.7 km north of the valley.
34. 96 ft **36. a.** (8, –13.9)
b. about 16 ft **38.** A

Lesson 7-4

2. The period is $\frac{2\pi}{4}$, or $\frac{\pi}{2}$, not $\frac{\pi}{4}$.
4. $-1 \le y \le 1$; The range is from minimum to maximum; the amplitude is half that distance. **6.** period: 4π; amplitude: $\frac{1}{2}$
8. period $\frac{1}{2}$: $y = \frac{1}{2}\cos(4\pi x)$;
period 2: $y = \frac{1}{2}\cos(\pi x)$;
period 4: $y = \frac{1}{2}\cos\left(\frac{\pi}{2}x\right)$



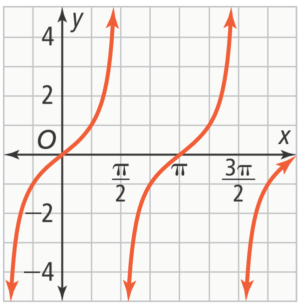
Selected Answers

Topic 7

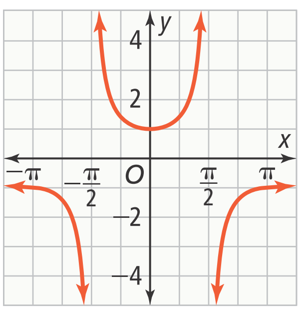
10. The student used the value of a instead of the value of b in the formula for the period of a function. The period is 3π. **12.** The graph of $y = \csc x$ is undefined where $y = 0$ on the graph of $y = \sin x$. **14.** amplitude: $\frac{1}{2}$; period: 16π **16.** frequency: $\frac{1}{\pi}$; average rate of change: varies between $\frac{-3}{\pi}$ and $\frac{3}{\pi}$, the average rate of change over the whole interval $[0, \pi]$ is 0. **18.** $y = 2 \sin \frac{\pi}{4}x$ **20. a.** 12 h 25 min **b.** 3 ft **c.** $D(t) = 3\cos\left(\frac{24\pi}{149}t\right) + 5$ **22.** A

Lesson 7-5

2. The period of the tangent function is π. **4.** The coefficient compresses the graph vertically by a factor of $\frac{1}{2}$.

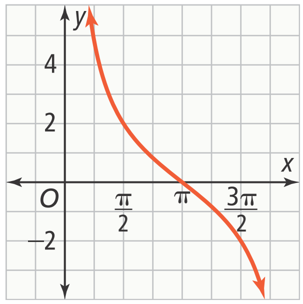


6. The range of sine and cosine is all real numbers from −1 to 1, inclusive; the range of cosecant and secant is all real numbers greater than or equal to 1 or less than or equal to −1. The values they all share are ± 1. **8.** The graph has been shifted up one unit. This is the correct graph for $y = \sec x$:



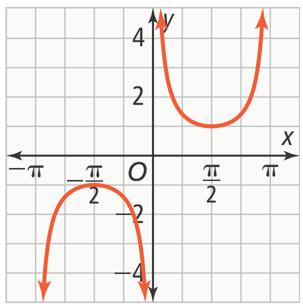
10. The function for $y = \tan x$ is undefined at all the odd integer multiples of $\frac{\pi}{2}$ or 90°. **12.** all the odd multiples (positive and negative) of $\frac{\pi}{2}$ **14.** Cosine and secant are even; all the others are odd. **16.** domain: $\{x : x \neq \frac{\pi}{2} + n\pi$, where n is an integer$\}$; range: $(-\infty, \infty)$; period: π; zeros: $\{x : x = n\pi$, where n is an integer$\}$ asymptotes: any multiple of $\frac{\pi}{2}$

18.



There is a vertical stretch that makes the graph look more straight than the parent function $y = \cot x$. The horizontal stretch changes the period of the function from π (for $y = \cot x$) to 2π.

20.



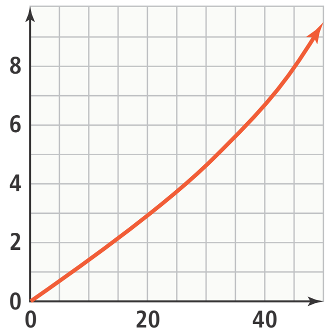
The domain of $y = \csc x$ is $\{x: x \neq n\pi$ where n is an integer$\}$. The graph has a vertical asymptote wherever $x = n\pi$, for some integer n, which is where the graph of $\sin x = 0$. The range of $\sin x$ is $\{x: 0 \leq x \leq 0\}$, whereas the graph of $y = \csc x$ approaches $+\infty$ on one side of the asymptote and $-\infty$ on the other side.

Selected Answers

Topic 7

22. a. $h = 8 \tan \theta$

b.



24. B

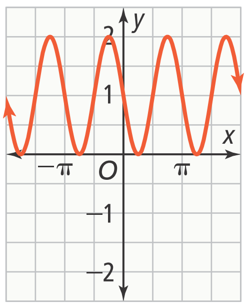
Lesson 7-6

2. A phase shift is a horizontal translation of a periodic function.

4. $y = \frac{1}{6} \sin \left[\frac{3}{4}(x - 2\pi)\right] + 5$

6. amplitude: $\frac{1}{3}$; period: π; phase shift: left $\frac{\pi}{2}$ units; vertical shift: down 1 unit

8.

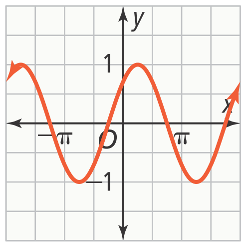


10. The student did not first factor out a 3. The phase shift is left $\frac{\pi}{6}$ units.

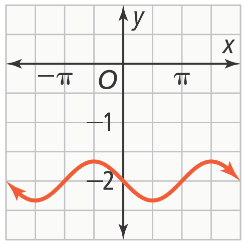
12. The graph of $y = \frac{1}{4} \cos\left[3\left(x - \frac{2\pi}{3}\right)\right] + 2$ is compressed by a factor of $\frac{1}{4}$, shifted right $\frac{2\pi}{3}$ units, and shifted up 2 units from the graph of $y = \cos x$. The horizontal shift does not affect the domain and range of the function. The compression factor divides the range interval by one-fourth and the vertical shift up 2 units adds 2 units to the compressed range interval.

14. The zeros of $y = \sin\left(x + \frac{\pi}{3}\right)$ are shifted left $\frac{\pi}{3}$ units of the zeros of $y = \sin x$.

16.



18.



20. amplitude: $\frac{2}{3}$; period: 2π; phase shift: left $\frac{\pi}{3}$ units; vertical shift: up 3 units; maximum: $3\frac{2}{3}$; minimum: $2\frac{1}{3}$

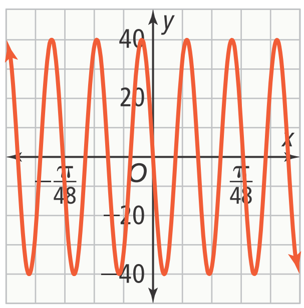
22. $y = \frac{1}{2} \sin \left(x - \frac{3\pi}{2}\right) + 1$

24. a. $V(t) = 40 \cos\left(188t + \frac{\pi}{2}\right)$

b. $V(t) = 40 \cos \left[188 \left(t + \frac{\pi}{376}\right)\right]$

c. 40 **d.** $\frac{\pi}{94}$ **e.** shift left $\frac{\pi}{376}$ units

f.



26. a. yes **b.** no **c.** no **d.** yes

Selected Answers

Topic 7

28. Part A Sample: $y = \sin(x + 2\pi)$
Part B Sample: $y = \sin(x - 2\pi)$
Part C The period of the sine function is 2π. **Part D** infinitely many; The sine function has a period of 2π, so the graph repeats on every interval of 2π. There are an infinite number of intervals of 2π, so there are an infinite number of equations that will map the sine function onto itself.

Topic Review

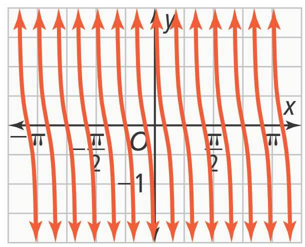
2. initial side **4.** amplitude
6. phase shift
8. $\sin\theta = \frac{6}{10}$; $\cos\theta = \frac{8}{10}$; $\tan\theta = \frac{6}{8}$;

$\csc\theta = \frac{10}{6}$; $\sec\theta = \frac{10}{8}$; $\cot\theta = \frac{8}{6}$

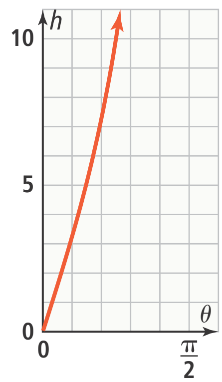
10. $\sin\theta = \frac{5}{13}$; $\cos\theta = \frac{12}{13}$; $\tan\theta = \frac{5}{12}$;

$\csc\theta = \frac{13}{5}$; $\sec\theta = \frac{13}{12}$; $\cot\theta = \frac{12}{5}$

12. cosecant **14.** $67°$ **16.** $155°$ **18.** $\frac{34\pi}{45}$
20. $\frac{4\pi}{9}$ **22.** $\frac{\pi}{4}$ in Quadrant III
24. For angle measures in radians, the arc length subtended by the angle in the unit circle is equal to the angle measure.
26. $\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$; $\cos\frac{4\pi}{3} = -\frac{1}{2}$
28. $\sin\frac{5\pi}{6} = \frac{1}{2}$; $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$
30. $\tan 120° = -\frac{\sqrt{3}}{2}$
32. $\sec -135° = -\sqrt{2}$; $\csc -135° = -\sqrt{2}$; $\cot -135° = 1$ **34.** $\sin\theta = -\frac{4}{5}$
36. amplitude: $\frac{1}{4}$; period: $\frac{\pi}{2}$; frequency: $\frac{2}{\pi}$ **38.** amplitude: 4; period: π; frequency: $\frac{1}{\pi}$ **40.** $y = 3\cos\frac{1}{2}x$
42. The graph is compressed vertically, which makes the graph look more straight than the parent function $y = \cot x$. The horizontal compression changes the period of the function from π to $\frac{\pi}{3}$.



44. $h = 10\tan\theta$; .



46. amplitude: 4; period: 2π; phase shift: 4π units to the left; vertical shift: 8 units down **48.** $y = 2\cos\left(x + \frac{\pi}{4}\right) + 1$

Selected Answers

Topic 8

Lesson 8-1

2. The inverse of $y = \cos(x)$ does not pass the vertical line test. In order to talk about the inverse as a function, the domain has to be restricted.
4. The student gave the measures of the angles for which the sine is 0. The angles whose sine is 1 are $\frac{\pi}{2} + 2\pi n$.

6. 60° or $\frac{\pi}{3}$ **8.** 56.1° + (360°)k, 123.9° + (360°)k, where k is an integer
10. $\theta = 124°$ and 304° **12.** The inverse relations would not be functions because there would be multiple y-values for the same x-value.
14. Cosine θ is never greater than 1.
16. a. $\frac{\pi}{8}$ **b.** –3.6 **c.** $-\frac{\pi}{2}$ **18.** Answers may vary. Sample: $2 \sin \theta + \sqrt{3} = 0$
20. 30° or $\frac{\pi}{6}$ **22.** 135° or $\frac{3\pi}{4}$ and –45° or $-\frac{\pi}{4}$ **24.** 39.8° + (360°)k, 140.2° + (360°)k, where k is an integer
26. –23.0° + (360°)k, –157.0° + (360°)k, where k is an integer **28.** $\frac{5\pi}{6}$, $\frac{11\pi}{6}$
30. $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$ **32.** about 1.03 minutes
34. a. 120 mi **b.** about 2.13 s
36. 1:46 A.M. **38.** B

Lesson 8-2

2. The Law of Sines can give zero, one, or two solutions. When solving for an unknown angle, if the solution is an acute angle, then you will need to use that answer to find a second angle. If there is no solution, the Law of Sines gives no valid solutions.

4. The Law of Sines is used to solve for a side or angle measure when you have a side and angle pair (a and A, for example), and one more side or angle. The Law of Cosines is used to find an angle when you have all three sides or to find a side when you have the other two sides and the included angle.
6. about 64.7° **8.** about 4.3 **10.** about 6.9 **12.** No, Lourdes is not correct. To use the Law of Sines, you need a corresponding side and angle, plus one other side or angle. If you have sides a and b, and angle C, for example, it would not work. **14.** Since cos 140° is negative, the student should have added 193.03 to 277 to get $a^2 = 470.03$. Then $a = 21.68$.
16.
$$c^2 = a^2 + b^2 - 2ab(\cos 90)$$
$$c^2 = a^2 + b^2 - 2ab(0)$$
$$c^2 = a^2 + b^2 - 0$$
$$c^2 = a^2 + b^2$$
This is the Pythagorean Theorem.
18. Draw the altitude from G, label its length x.

$\sin E = \frac{x}{f}$ and $\sin F = \frac{x}{e}$

$f \sin E = x$ and $e \sin F = x$

$f \sin E = e \sin F$

$\frac{\sin E}{e} = \frac{\sin F}{f}$

20. about 23.5° **22.** one: $m\angle Y \approx 16.1°$
24. zero **26.** about 33.6°
28. about 73.1° **30.** about 79.9°
32. about 43.2 yd **34.** about 40.1°
36. B

Selected Answers

Topic 8

Lesson 8-3

2. Because $\sin(-x) = -\sin x$, the sine function is odd; however since $\cos(-x) = \cos x$, the cosine function is an even function. **4.** The cofunction identities were derived using the unit circle, which defines the trigonometric ratios for every real-number angle measure.

6. $\sin\theta \bullet \sec\theta \bullet \cot\theta \stackrel{?}{=} 1$

$$\sin\theta \bullet \frac{1}{\cos\theta} \bullet \frac{\cos\theta}{\sin\theta} \stackrel{?}{=} 1$$

$$\cancel{\sin\theta} \bullet \frac{1}{\cancel{\cos\theta}} \bullet \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}} \stackrel{?}{=} 1$$

$$1 = 1 \ \checkmark$$

8. $\sec\theta$ **10.** $\frac{\sqrt{6}-\sqrt{2}}{4}$ **12.** Select one side of the equation and transform it until it is the same as the other side of the equation.

14. $\tan(-x) = \frac{\sin(-x)}{\cos(-x)}$

$$= -\frac{\sin x}{\cos x}$$

$$= -\tan x$$

16. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{x}{r} \bullet \frac{r}{y} = \frac{x}{y}$

18. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

20. $\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\csc\theta$; therefore cosecant is an odd function.

22. $\cot(-\theta) = \frac{1}{\tan(-\theta)} = \frac{1}{-\tan\theta} = -\cot\theta$; therefore cotangent is an odd function.

24. $-\cos x$

26. $\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$

$$= \frac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= \frac{\frac{\sin\alpha\cos\beta - \sin\beta\cos\alpha}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\beta\cos\alpha}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}}{1 + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

28. $\frac{-\sqrt{6}-\sqrt{2}}{4}$; -0.966 **30.** $\frac{\sqrt{2}+\sqrt{6}}{4}$; 0.966

32. $y = 2\sin(1{,}320\pi x)$

34. $F = Mg\sin\theta\sec\theta$

36. a. $\cos^2 x$ **b.** $\sin^2 x + 2\cos^2 x$

38. Part A $\theta = 13.3°$

Part B $\sin\theta = \frac{v^2\cos\theta}{gR}$

Lesson 8-4

2. To find a complex conjugate, reflect over the real x-axis. Reflecting over the real x-axis affects only the y-coordinate, or the imaginary part of the complex number. Casey took the opposite of both parts of the complex number. The complex conjugate of $7 + 3i$ is $7 - 3i$.

Selected Answers

Topic 8

4. Opposite sides of a parallelogram are congruent. The side formed by the two points and the side that is the modulus of the difference are opposite sides when graphed as a parallelogram, so they must be the same length.
6. (–6, 2) **8.** $\sqrt{41}$ **10.** –4.5 – 0.5i
12. No, LaTanya is not correct. The slope of $\overline{RP}$ is $\frac{-9}{3}$. This reduces to –3, but she needed to count down 9 and right 3 from point S, not down 3 and right 1. The point T should be at (8, 2).
14. Using the Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$$

$d = \sqrt{(x_2)^2 + (y_2)^2}$ Using the modulus formula gives:

$$\text{modulus} = \sqrt{(a + bi)(a - bi)}$$

$$\text{modulus} = \sqrt{a^2 + abi - abi - (bi)^2}$$

$$\text{modulus} = \sqrt{a^2 - b^2 i^2}$$

$$\text{modulus} = \sqrt{a^2 - b^2(-1)}$$

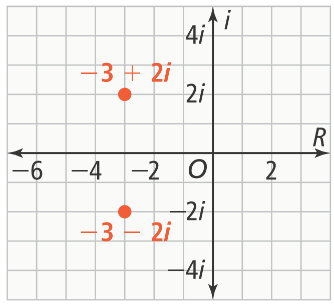
$$\text{modulus} = \sqrt{a^2 + b^2}$$

Since $x_2 = a$ and $y_2 = b$,

$$\sqrt{(x_2)^2 + (y_2)^2} = \sqrt{a^2 + b^2}$$

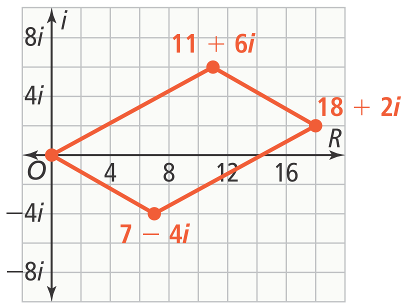
16. Using a parallelogram with the given complex numbers models addition only. In order to model subtraction, you have to write the subtraction problem as adding a negative, or the opposite, instead.

18.



20. (2, –2) **22.** $\sqrt{340} \approx 18.44$

24.



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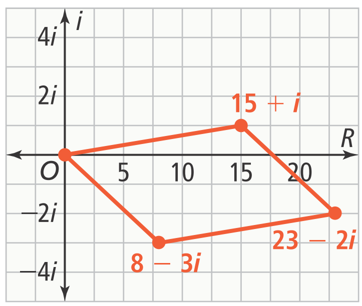


28. $\sqrt{265} \approx 16.28$ **30.** $\sqrt{157} \approx 12.53$
32. 9.2 + 0.8i ohms **34. I.** 0.5 – 8.5i
II. 4 + 14i **III.** –11 + 8i **36. Part A** 4 + i
Part B –2 + 3.5i and 2 + 3.5i **Part C** The segments have lengths $\sqrt{41}$ units, $\sqrt{41}$ units, and 4 units.

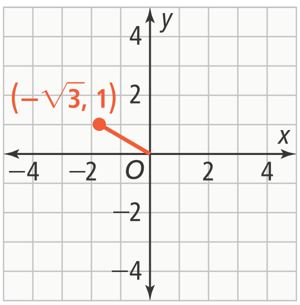
Selected Answers

Topic 8

Lesson 8-5

2. Lucas subtracted $\frac{7\pi}{4}$ and $\frac{3\pi}{4}$ instead of adding them. The correct answer is 6 cis $\frac{5\pi}{2}$. **4.** Answers may vary. Sample: The argument of a complex number tells you its direction from the origin in the complex plane. **6.** $10\sqrt{2}$ cis $\frac{3\pi}{4}$
8. $-3 + 3\sqrt{3}\,i$ **10.** 21 cis $\frac{3\pi}{4}$ **12.** 21 cis $\frac{5\pi}{3}$
14. $-8i$ **16.** Use the formula $z^n = r^n$ cis $n\theta$. **18.** Draw a segment 3 units long from the origin at an angle of $\frac{5\pi}{6}$ with the real axis. **20.** Both points are 4 units from the origin and since $-\frac{\pi}{6}$ and $\frac{11\pi}{6}$ differ by a multiple of 2π, the points are at the same location.

22.



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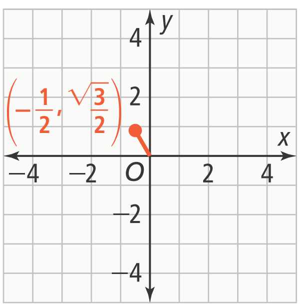


26. $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ **28.** $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$
30. 3.61 cis 5.30 **32.** 4.24 cis $\frac{5\pi}{4}$
34. 24 cis π; -24

36. $z_1 = 2$ cis $\frac{\pi}{6}$; $z_2 = 4$ cis $\frac{5\pi}{3}$; $z_1 z_2 = 8$ cis $\frac{11\pi}{6}$; $4\sqrt{3} - 4i$

38. 2,025 cis −4.43; $-567 + 1{,}944i$

40. 2 cis $\frac{5\pi}{6} = -\sqrt{3} + i$ amps

42. [$2\sqrt{2}$] [–][$2\sqrt{2}$]

44. Part A

$$d = \sqrt{(r_1)^2 + (r_2)^2 - 2r_1 r_2\cos(\theta_2 - \theta_1)}$$

Part B Yes, you get the same formula:

$$d = \sqrt{(r_1)^2 + (r_2)^2 - 2r_1 r_2\cos(\theta_2 - \theta_1)}$$

Part C 7.74

Topic Review

2. trigonometric identity **4.** real axis
6. modulus of a complex number
8. 60° or $\frac{\pi}{3}$ **10.** −45° or $\frac{-\pi}{4}$ **12.** $\frac{\pi}{6}, \frac{7\pi}{6}$
14. The domain of inverse sine must be equal to the range of the sine function, or [−1, 1]. **16.** 23.4° **18.** 43.5° **20.** Use Law of Sines when you know the measures of two angles of a triangle and a side opposite one of the angles or you know the measures of two sides and an angle opposite one of the sides. When you know the measures of two sides and their included angle, use Law of Cosines.

22. $\sec^2 x$ **24.** $\frac{1}{\cos^2 x - \sin^2 x}$ **26.** $\frac{\sqrt{6} - \sqrt{2}}{4}$
28. $-2 - \sqrt{3}$ **30.** $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

32. $(-2, -5i)$ **34.** $\sqrt{233}$ **36.** $3\sqrt{5}$

Selected Answers

Topic 8

38. They are equal. The modulus of the complex number z is equal to the square root of the product of z and its conjugate $\overline{z}$. Since the complex conjugate of the complex conjugate of z is z, the modulus of $\overline{z}$ is equal to the same expression.

40. $\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

42. 5.83 cis 1.03 **44.** $2\sqrt{2}+i$ **46.** 2 cis $\frac{\pi}{4}$; Answers may vary. Sample: You take the square root of the modulus and half the argument to take the square root of a complex number in polar form.

Selected Answers

Topic 9

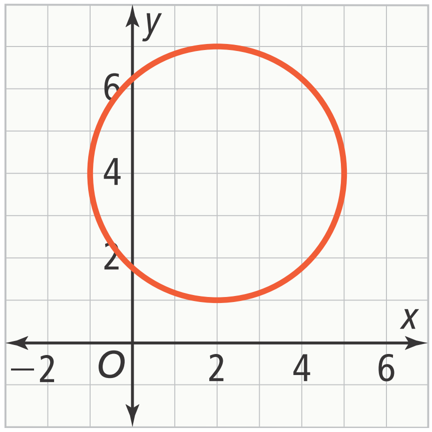
Lesson 9-1

2. A conic section is the curved shape that results from taking a cross-section of a double right cone. **4.** The parabola opens downward and the vertex is (0, 0). Since $a = -4$, $c = -\frac{1}{16}$, the focus is $\left(0, -\frac{1}{16}\right)$ and the directrix is $y = \frac{1}{16}$. **6.** $y = \frac{1}{8}x^2$ **8. a.** (0, 0) **b.** $y = ax^2$ **c.** 3 **d.** $y = \frac{1}{12}x^2$ **10.** $y = \frac{1}{16}x^2$ **12.** The student wrote a correct equation, but interpreted incorrectly. The vertex is (0, 4), not (4, 0), and since the parabola opens left and $c = -\frac{1}{2}$, the directrix is $x = \frac{1}{2}$, not $x = -2$. **14.** The y-intercept is $\left(0, \frac{1}{3}\right)$. **16.** $y = -\frac{1}{8}x^2$ **18. a.** (0, 0) **b.** $x = ay^2$ **c.** focal length = 5 **d.** $x = -\frac{1}{20}y^2$ **20.** (0, 40) **22. a.** $x = -2$ **b.** 10 units **24. a.** Answers may vary. Sample: $y = \frac{1}{48}x^2$ **b.** 3 feet **26.** C

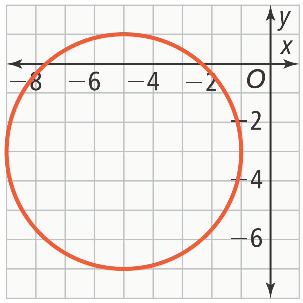
Lesson 9-2

2. Given the center (h, k) and the radius r, substitute values into the standard form of the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$. **4.** Use the Midpoint Formula to find the midpoint of the diameter. This point is the center of the circle. **6.** (–3, –7); 7 **8.** (9, 4); $\sqrt{11}$
10. $(x + 3)^2 + (y - 9)^2 = 16$ **12.** 17π
14. $(x + 4)^2 + (y - 5)^2 = 12$
16. $(x - 4)^2 + (y - 4)^2 = 16$
18. $x^2 + y^2 = 2.2^2$

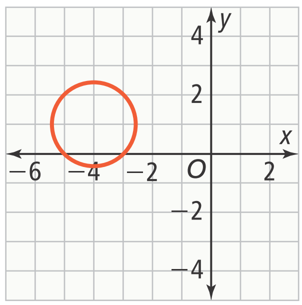
20. $(x - 2)^2 + (y - 4)^2 = 9$;



22. $(x + 5)^2 + (y + 3)^2 = 16$;



24. $(x + 4)^2 + (y - 1)^2 = \frac{9}{4}$;



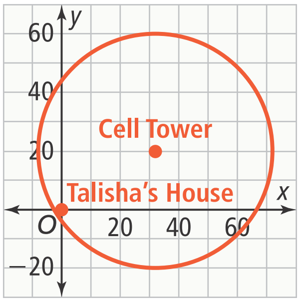
26. The equation of this circle in standard form is $(x + 3)^2 + (y + 2)^2 = 4$. The circle has center (–3, –2) and radius 2. **28.** The equation of this circle in standard form is $(x - 6)^2 + (y + 4)^2 = 49$. The circle has center (6, –4) and radius 7.
30. (2, 2) and (–2, –2)
32. (3, –4) and (–4, –3)
34. $(x - 5)^2 + (y + 3)^2 = 25$

Selected Answers

Topic 9

36. a. (32, 20)
b. $(x - 32)^2 + (y - 20)^2 = 1{,}600$

c.



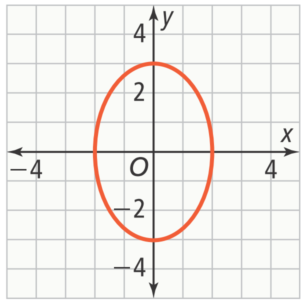
d. Yes; the graph of the circle, which represents the range of the cell phone tower, includes Talisha's house.

38. D

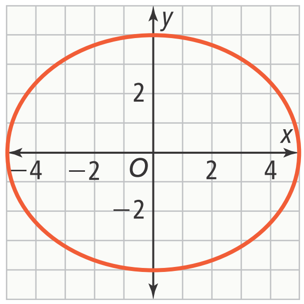
Lesson 9-3

2. The co-vertices are the endpoints of the minor axis. **4.** The greatest measure is *a,* the distance from the center to one vertex on the major axis. The distance *b* is from the center to one of the co-vertices, which is on the shorter minor axis, and *c* is the distance from the center to one of the foci, which is along the major axis but in the interior of the ellipse, so it is shorter than *a*. **6.** vertices: (–3, –5) and (5, –5); co-vertices: (1, –3) and (1, –7)
8. $(-5 + 2\sqrt{3}, 9)$ and $(-5 - 2\sqrt{3}, 9)$, or approximately (–1.5, 9) and (–8.5, 9)
10. $Y1 = \frac{\sqrt{36 - x^2}}{3}$ and $Y2 = -\frac{\sqrt{36 - x^2}}{3}$
12. a. vertical **b.** (–5, 0) and (5, 0)
c. (0, –7) and (0, 7) **14.** No; the vertices of an ellipse are the points farther away from the center, and the co-vertices are the points closer to the center.

16.



18.



20. $\frac{x^2}{36} + \frac{y^2}{121} = 1$ **22.** $\frac{x^2}{900} + \frac{y^2}{275} = 1$; about 33 inches **24. a.** The ellipse is horizontal. The major axis is 60 feet long, so $2a = 60$. Therefore, $a = 30$ feet. **b.** The ellipse is horizontal. The minor axis is 30 feet long, so $2b = 30$.
Therefore, $b = 15$ feet.
c. $\frac{x^2}{900} + \frac{y^2}{225} = 1$
26. about 41.5 m **28.** B

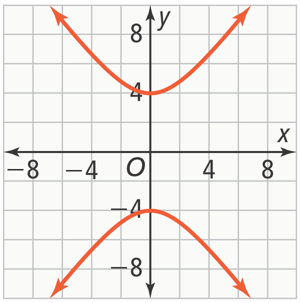
Lesson 9-4

2. While one side of a hyperbola appears to have the same shape as a parabola, the relationships that define the points on a hyperbola are different from those that define the points on a parabola.
4. The asymptotes are lines that intersect. The graph of a hyperbola approaches, but never intersects, the asymptotes. By drawing the asymptotes on a graph, you can use them as guidelines to sketch the hyperbola between them, making sure the hyperbola approaches, but never intersects, the asymptotes.

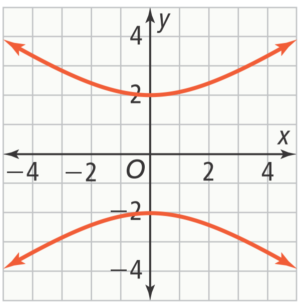
Selected Answers

Topic 9

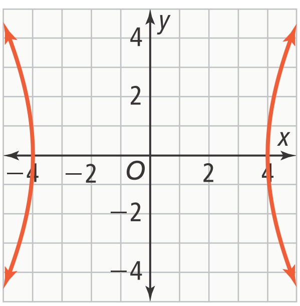
6. (0, 12) and (0, –12)
8. (10, 0) and (–10, 0)
10. $y = \pm\frac{10}{3}x$ **12.** A focus at (0, 1) means the value of c is 1, so $c^2 = 1$. A vertex at (0, 0.5) means the value of a is 0.5, so $a^2 = 0.25$. Use the equation $a^2 + b^2 = c^2$ to find the value of b: $0.25 + b^2 = 1$, so $b^2 = 0.75$ and $b = \sqrt{0.75}$.
14. $\frac{y^2}{16} - \frac{x^2}{9} = 1$



16. $\frac{x^2}{20} - \frac{y^2}{58} = 1$
18. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
20.



22.

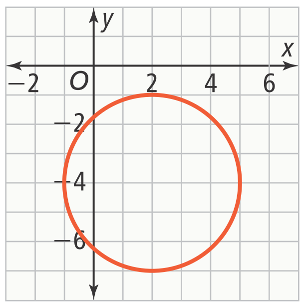


24. $\frac{y^2}{36} - \frac{x^2}{81} = 1$ **26.** $\frac{y^2}{25} - \frac{x^2}{144} = 1$; The focus is 8 units behind the mirror.

28. hyperbola **30.** circle **32** $\frac{y^2}{4} - \frac{x^2}{2.25} = 1$
34. $\frac{y^2}{1} - \frac{x^2}{3} = 1$ **36. Part A** For x-values of 0, 1, 2, and 3, the expression $x^2 - 16$ is negative. The square root of a negative number results in entries that show ERROR. **Part B** The table shows that (4, 0) is a vertex of the equation $y^2 - x^2 + 16 = 0$. This means (–4, 0) is also a vertex.
Part C $\frac{x^2}{16} - \frac{y^2}{16} = 1$; The value of a^2 is 16, so $a = 4$. This means the vertices are at (–4, 0) and (4, 0).

Topic Review

2. transverse axis **4.** ellipse **6.** parabola
8. $x = -\frac{1}{16}y^2$ **10.** A parabola opens toward the focus. **12.** $(x - 2)^2 + (y + 4)^2 = 9$



14. $2\sqrt{23}$ **16.** $\frac{x^2}{73} + \frac{y^2}{9} = 1$
18. Write the second-degree equation in general form $Ax^2 + Cy^2 + Dx + Ey + F = 0$. If A or C (but not both) is zero, then the equation represents a parabola. If the A and C are equal, then the equation represents a circle. If A and C are both positive, but different, then the equation represents an ellipse. If A and C have opposite signs, then the equation represents a hyperbola.

Selected Answers

Topic 10

Lesson 10-1

2. Her answer should be $\begin{bmatrix} 0 & 0 \\ -8 & 0 \end{bmatrix}$, since $-4 - 4 = -8$ and not 0. **4.** Equal matrices have the same dimensions, and their corresponding elements are equal. Sample: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **6.** −6

8. $\begin{bmatrix} -3 & 9 \\ -11 & 11 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & -9 \\ 11 & -11 \end{bmatrix}$

12. You would set up equations for corresponding elements and then solve for the variable; $a = 4$, $b = 0$, $c = 6$, $d = -1$ **14.** In the translation matrix, Row 2 should be all 1s since the points are being translated up 1 unit. The answer matrix should be $\begin{bmatrix} -2 & -1 & -6 \\ -2 & 2 & -1 \end{bmatrix}$.

16. Matrices A and B have the same corresponding elements when they are simplified. **18.** The area of the new square is 9 times the area of the original square. **20.** $\begin{bmatrix} 2.10 & 2.80 & 3.50 \\ 1.75 & 2.45 & 3.85 \end{bmatrix}$

22. $\begin{bmatrix} -7 & 1 & 6 \\ -3 & 1 & 0 \end{bmatrix}$ **24.** $\begin{bmatrix} 7 & -1 & -6 \\ 3 & -1 & 0 \end{bmatrix}$

26. $\begin{bmatrix} -2 & 0 \\ -6 & 5 \\ 4 & -11 \end{bmatrix}$ **28.** $\begin{bmatrix} -9 & 1 \\ -4 & -10 \\ -3 & 7 \end{bmatrix}$

30. $U(20, -4)$; $V(24, 44)$

32. a. $\begin{bmatrix} 0.05 \\ 0.11 \\ 0 \\ 6.07 \end{bmatrix}$ **b.** $\frac{1}{60}B$ **34. C**

36. Part A after 2 seconds:

$$\begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 200 & 200 & 200 \end{bmatrix} = \begin{bmatrix} 100 & 150 & 200 \\ 250 & 350 & 250 \end{bmatrix}$$

after 5 seconds:

$$\begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 500 & 500 & 500 \end{bmatrix} = \begin{bmatrix} 100 & 150 & 200 \\ 550 & 650 & 550 \end{bmatrix}$$

Part B after 3 seconds:

$$\begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix} + \begin{bmatrix} 150 & 150 & 150 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 250 & 300 & 350 \\ 50 & 150 & 50 \end{bmatrix}$$

after 8 seconds:

$$\begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix} + \begin{bmatrix} 400 & 400 & 400 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 500 & 550 & 600 \\ 50 & 150 & 50 \end{bmatrix}$$

Part C

$$\begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix} + n\begin{bmatrix} 50 & 50 & 50 \\ 0 & 0 & 0 \end{bmatrix}$$

for $n = 1$ to 16

Lesson 10-2

2. Because the number of columns in the first matrix, 5, is not equal to the number of rows in the second matrix, 4
4. The student is not correct. There is only one element. **6.** $\begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 \\ -3 & -1 & 1 \end{bmatrix}$

10. If the matrices were not written in the given order, the number of columns in the first matrix would not equal the number of rows in the second matrix.

Selected Answers

Topic 10

12. a. a reflection across the *x*-axis
b. a 90° clockwise rotation about the origin

14. Yes; they both equal $\begin{bmatrix} -1 & -7 \\ 2 & -3 \end{bmatrix}$.

16. $IQ = \begin{bmatrix} 1 & -3 & 2 \\ -4 & 5 & -6 \\ 9 & -7 & 8 \end{bmatrix}$

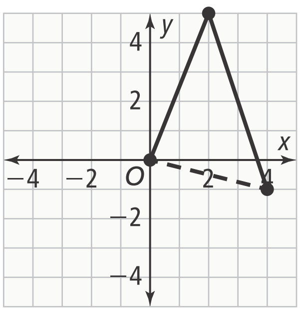
18. $\begin{array}{r} \text{Snack Bar A} \\ \text{Snack Bar B} \\ \text{Snack Bar C} \end{array} \begin{bmatrix} 435 \\ 430 \\ 345 \end{bmatrix}$

20. a. $W = [0.45 \;\; 0.30 \;\; 0.25]$
b. $WG = [93.2 \;\; 87.25 \;\; 86.15]$ **22.** D

Lesson 10-3

2. She mistakenly added 11 and 2, rather than 11 and –4; $\overrightarrow{AC} = \langle 7, 7 \rangle$
4. Since a vector can be represented as a 2×1 matrix, its components can be transformed by matrix multiplication.
6. $\langle -3, -7 \rangle$ **8.** $\langle -3, -11 \rangle$; magnitude ≈ 11.4; direction ≈ 254.7° **10.** sum $\langle 3, 10 \rangle$; difference $\langle 9, 10 \rangle$

12. Rather than placing the beginning of $\overrightarrow{BC}$ at the terminal point of $\overrightarrow{AB}$, the student placed it at the origin.



14. No; even though they have the same magnitude, their directions are opposites. **16.** The magnitude of $\vec{v}$ is approximately 24 kg. **18.** $\langle -1, -4 \rangle$; magnitude $= \sqrt{17} \approx 4.1$; direction ≈ 256°

20. $\langle -1, 10 \rangle$; magnitude $= \sqrt{101} \approx 10.05$; direction ≈ –84.3° or 95.7° **22.** $\langle -8, 19 \rangle$
24. The magnitude of her boat is about 15.3 mph in the direction of about 236.2°. **26.** $\langle -2, 9 \rangle$; magnitude ≈ 9.2; direction ≈ 102.5° **28.** $\langle -28, -36 \rangle$
30. $\langle 12, -2 \rangle$ **32.** horizontal: 17.3 mph; vertical: 10 mph **34.** magnitude 60.8 lb; $\langle 59.8, 10.6 \rangle$ **36. D**

Lesson 10-4

2. For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant of *A*, denoted det *A*, is the value $ad - bc$.
4. The area of the triangle defined by vectors $\langle a, b \rangle$ and $\langle c, d \rangle$ and the matrix $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{2}|\det T|$. **6.** The inverse does not exist.

8. $\begin{bmatrix} \frac{5}{2} & -\frac{2}{3} & 4 \\ -\frac{1}{2} & \frac{1}{3} & -1 \\ 1 & -\frac{1}{3} & 2 \end{bmatrix}$

10. 48 square units **12.** No; only square matrices, with the same number of rows and columns, have an inverse. Because a 2×3 matrix is not a square matrix, it cannot have an inverse.
14. Yes; when the product of two matrices equals the identity matrix, the matrices are inverses. Because $\begin{bmatrix} 8 & 4 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the matrices are inverses. **16.** Monisha is not correct because subtraction is not commutative.

18. $\begin{bmatrix} \frac{3}{20} & \frac{1}{10} \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix}$ **20.** $P^{-1} = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$

Selected Answers

Topic 10

22. Q^{-1} does not exist. **24.** MATRICES ARE FUN **26.** 10 square units **28.** 16 square units **30.** MATHEMATICS EDITOR **32.** 100,000 square feet **34.** D

Lesson 10-5

2. The coefficient matrix is $\begin{bmatrix} 3 & 2 & 0 \\ 0 & -1 & 4 \\ 2 & 0 & 6 \end{bmatrix}$, with an entry for each variable in each row.

4. Express the system as a matrix equation. Find the inverse of the coefficient matrix. Multiply both sides of the matrix equation by the inverse matrix. Multiplying the coefficient matrix by its inverse gives the identity matrix. Multiplying the inverse matrix by the constant matrix gives the solution matrix.

6. $\begin{bmatrix} 6 & -8 & 2 \\ -1 & 5 & 3 \\ 9 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} -46 \\ 29 \\ -35 \end{bmatrix}$

7. $A^{-1}\begin{bmatrix} \frac{2}{7} & \frac{27}{70} & \frac{1}{70} \\ -\frac{1}{7} & -\frac{17}{70} & -\frac{11}{70} \\ -\frac{2}{7} & -\frac{3}{35} & -\frac{4}{35} \end{bmatrix}$; $x = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

8. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & -3 \\ 2 & -13 & 9 \\ -4 & 12 & -6 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$

10. Sample: Solving a system of linear equations using substitution or elimination usually requires more calculations then solving a system of linear equations using matrices.

12. When there are infinitely many solutions, $e = f$. When there are no solutions $e \neq f$. **14.** $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

16. $\begin{bmatrix} 8 & 1 \\ -12 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

18. no solution **20.** $x = 8$, $y = 5$

22. 2 hr to make one bracelet and 3 hr to make one necklace

24. $\begin{bmatrix} \frac{1}{3} & 1 & 2 \\ \frac{2}{3} & \frac{4}{5} & 1 \\ 1 & 2 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 22 \\ 37 \end{bmatrix}$

swimming: 3 mph; biking: 15 mph; running: 8 mph

26. B **28. Part A** $\begin{cases} 2a = 1 \\ 2b = 3 \end{cases}$

Part B $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$

Part C $N_2 + 3\,H_2 \rightarrow 2NH_3$, balanced 2N and 6H on each side.

Part D $4Cr + 3O_2 \longrightarrow 2Cr_2O_3$

Topic Review

2. variable matrix **4.** scalar multiplication **6.** inverse matrix **8.** square matrix

10. $\begin{bmatrix} -35 & 5 \\ 40 & 10 \end{bmatrix}$ **12.** Matrix P will have the same dimensions as matrix N, 3×3. Matrix P will have the elements that are the opposites of the corresponding elements in matrix N.

14. $\begin{bmatrix} -13 & 29 \\ -45 & 9 \end{bmatrix}$ **16.** $\begin{bmatrix} -26 & -9 \\ -28 & 3 \end{bmatrix}$

18. $\begin{bmatrix} 24 & -27 \\ 8 & -24 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 & -3 \\ 4 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -3 \\ -4 & -1 & 0 \end{bmatrix}$

22. $\begin{bmatrix} 800 & 500 \\ 750 & 550 \end{bmatrix}\begin{bmatrix} 25 \\ 20 \end{bmatrix} = \begin{bmatrix} 30{,}000 \\ 29{,}750 \end{bmatrix}$; \$30,000 at Store X, \$29,750 at Store Y **24.** $\langle 4, 4 \rangle$; $\langle -18, 14 \rangle$ **26.** $\overline{MN}$ is rotated 180°. **28.** 12

30. $\begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$

Selected Answers

Topic 10

32. Carla used $A^{-1} = \frac{1}{\det A}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to find the inverse instead of $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

The inverse is $\begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & -2 \end{bmatrix}$.

34. $x = 6$, $y = -2$

36. The first column in the coefficient matrix represents the coefficients of x. The second column in the coefficient matrix represents the coefficients of y. The third column in the coefficient matrix represents the coefficients of z. The values in the matrix on the right side of the equation represent the numbers on the right side of the equal sign for three equations in the system.

The first row in the coefficient matrix represents the first equation in the system, so the first equation is $4x + 9y + z = 3$. The second row in the coefficient matrix represents the second equation in the system, so the second equation is $8x - 2y = 10$. The third row in the coefficient matrix represents the third equation in the system, so the third equation is
$-7x + 3y + 2z = 6$.

The system is: $\begin{cases} 4x + 9y + z = 3 \\ 8x - 2y = 10. \\ -7x + 3y + 2z = 6 \end{cases}$

37. \$1 for each folder and \$5 for each notebook

Selected Answers

Topic 11

Lesson 11-1

2. The survey was conducted on a qualitative, or categorical, variable, rather than a quantitative variable.
4. The population is the 1,560 students that attend Hana's high school.
6. The statistical variable is the number of TV sets owned by families. It is quantitative because it asks for an amount, rather than a brand or type of TV set.
8. Yes; this is a scatter plot that represents the answers given by students of different ages to a question about the number of hours spent studying.
10. The population should be all the residents of the city of Detroit, since it is a city tax, rather than a state tax.
12. a. No; it is a question to which there is only one answer. **b.** Yes; this is a question to which multiple responses are anticipated. **14. a.** The sample is the 40 students the biology teacher randomly selected to participate in his survey. **b.** The population is all of the students in all 5 of this biology teacher's classes. **16. a.** Yes; it is a statistical question that can be answered by collecting the responses of many people. **b.** No; it is a question for which there is only one answer.
18. This is a quantitative statistical question because the responses are numerical in nature and can be used in calculations. **20.** parameter
22. Part A The population is high schools in the state. The sample is the list of high schools that were surveyed. This data are quantitative, because the percents reported can be compared.
Part B Sample: What percent of students in a high school attend home football games?
Part C Joshua may have been surprised that the median values were so high because, in his experience, student attendance at home high school football games is considerably lower.

Lesson 11-2

2. bias **4.** sample survey **6.** The method is biased because the doctor is using patients' medical records to assign them to a group rather than assigning them randomly. **8.** Control group—people with psoriasis given a placebo; Experimental group—people with psoriasis given the new medication; Everyone's medication looks and feels the same, so no one knows whether the cream is a placebo or medication. **10. a.** The manager could randomly ask several employees within each department. **b.** The manager could conduct a survey of every 10th employee in the alphabetized employee list. **12.** Yes; he sampled plants from only one tray, so the sample is not random.
14. Randomly assign people age 65 and older to either the control group or the experimental group. The experimental group will participate in warm water therapy and the control group will not. Bias may result because the control group could be aware that they are not receiving any treatment. There is no placebo for warm water therapy.
16. Control group—use the same growing techniques as before; Experimental group—use the new fertilizer

Selected Answers

Topic 11

18. a. Since it says the proposed site of an “upscale” shopping center, the developer wants to be sure the people in the surrounding area will have enough money to spend at this type of venue. **b.** The market research company will have to use demographic data provided by the federal government.

20.

	Yes	No
A manager surveys every fourth customer about their level of satisfaction with their shopping experience.	❑	☑
When a school district wishes to get feedback on the district’s new webpage, they survey the entire population of randomly selected schools.	❑	☑
When Sheila wanted to find out what type of music was most popular among the students in her history class, she asked the two students who sat on either side of her.	☑	❑
The quality control officer of a ladder manufacturer walked into the shop, pulled the five closest ladders, and gave them several stress tests checking for potential defects.	☑	❑

22. Part A: The method was systematic sampling. Yes; it does seem valid. **Part B:** Based upon the results of the survey, it can be predicted that about 75% of the store’s customers would favor the expansion. **Part C:** categorical

Lesson 11-3

2. The graph of a model that is normally distributed has a bell-shaped curve that is symmetric about the mean.
4. The data distribution is skewed left because the long tail of data values is on the left side of the distribution.
6. mean: 6.3; standard deviation: ≈ 2.63; minimum: 2.25; 1st quartile: 3.9; median: 6.45; 3rd quartile: 8.8; maximum: 10.5
8. The data distribution is skewed right (positive) because the long tail of the data values is on the right side of the distribution. **10.** Maurice is correct that the distribution is skewed right. Maurice incorrectly concluded that the mean is less than the median. The mean is greater than the median.
12. mean: 28.4; standard deviation: 13.0; minimum: 9; first quartile: 17; median: 27; third quartile: 39; and maximum: 50
14. mean: 17.6; standard deviation: 8.5; minimum: 5; first quartile: 10; median: 17; third quartile: 24; and maximum: 33
16. Skewed right, so median and interquartile range best represent the data set; median: 3.7; interquartile range: 1.4 **18.** skewed left
20. skewed right **22.** skewed left; median = 17.5; interquartile range = 8.5
24. skewed right; median = 9.1; interquartile range = 9.45 **26. a.** mean: 86.79; standard deviation: 10.74
b. minimum: 60; first quartile: 80; median: 90.5; third quartile: 95; and maximum: 98 **c.** Because there is a long tail of data to the left, the distribution is skewed left. **d.** The test was an easy test because more students received higher scores, leading to a distribution that is skewed left.
28. This distribution is skewed right. That means that the median price will be less than the mean, so the real estate agent should report the mean to encourage her client to increase the asking price. **30.** D

Selected Answers

Topic 11

Lesson 11-4

2. The z-score counts how many standard deviations a data value is from the mean, because it divides the distance from the mean by the distance of a standard deviation. **4.** 86.4 **6.** –1.5 **8.** To verify the Empirical Rule for 68%, subtract the percentile for a z-score of –1 from the percentile for a z-score of 1: (84.1345) – (15.8655) = 68.269 ≈ 68%. To verify it for 95%, subtract the percentile for a z-score of –2 from the percentile for a z-score of 2: (97.725) – (2.275) = 95.45 ≈ 95%. To verify it for 99.7%, subtract the percentile for a z-score of –3 from the percentile for a z-score of 3: (99.865) – (0.135) = 99.73 ≈ 99.7% **10.** If the standard deviation of the scores on the English test were less than half the standard deviation of the scores on the French test, her z-score would be greater on the English test. For example, if the standard deviation of the English test scores was 2 and the standard deviation of the French test scores was 8, then Skyler's test score of 88 on the English test would be 2 standard deviations above the mean, and Skyler's test score on the French test would be 1 standard deviation above the mean. **12. a.** –2 or 2 **b.** Answers may vary. Sample: If the mean or standard deviation changes, Tyler's cost might have a different z-score. **14.** greater than 65,000 miles **16.** 2.5%

18. Holly scoring 25 points represents $z = 2.5$, and Juanita scoring 16 points represents $z = 3.2$; Juanita scored better relative to her average. **20.** 0.8944, or 89.44% **22.** 0.6844, or 68.44% **24.** 0.0119, or 1.19% **26.** –2 **28.** 0.6 **30. a.** mean: 739.4; standard deviation: 62.16 **b.** $\frac{4}{15}$ or 26.7% of the games had at least 808 people in attendance. **c.** z-score: $z = \frac{808 - 739.4}{62.16} \approx 1.104$, 13.48% of games would have had at least 808 people in attendence if the data were normally distributed.

32.

Interval	Probability (%)
at most 43	93.69%
at least 48	0.66%
between 32 and 38	43.38%
at least 41.6	10.35%
between 30.2 and 42.6	74.75%
at most 36.25	59.44%

34. Part A mean: 75; standard deviation: 7.86; first quartile: 74; third quartile: 80; interquartile range: 6 **Part B** The product of 1.5 and the interquartile range is 1.5(6) = 9°F. The data value 49°F is more than 9°F below the first quartile. So, 49°F is an outlier. **Part C** The data value 49°F is more than 3 standard deviations, or 3(7.86) = 23.58°F, below the mean. So, 49°F is an outlier.

Lesson 11-5

2. The teacher used the mean for categorical data, not for quantitative data. The margin of error should have been 1.

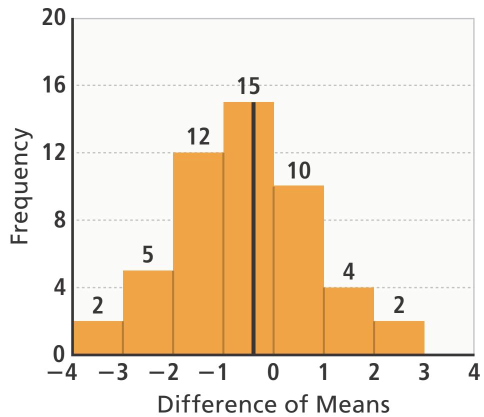
Selected Answers

Topic 11

4. Use the formula Margin of Error $= \frac{2\sigma}{\sqrt{n}}$ when given quantitative data. Use the formula Margin of Error $= \frac{1}{\sqrt{n}}$ when given categorical data. **6.** 4%; ±3% **8.** ±8.4 **10.** Tripling a sample size means that the margin of error is multiplied by $\frac{1}{\sqrt{3}}$, or about 0.58. This means the margin of error is about 58% of its original value. **12.** The student should have used the formula Margin of Error $= \frac{2\sigma}{\sqrt{n}}$ because the standard deviation is given. The range of reasonable means is between 70.2 and 71.8 inches. **14.** 30 min; 30% **16.** 95% of the mean number of successes from 50 trials fell between 75% and 85.9%. **18. a.** ±2% **b.** 5% to 9% **20. a.** mean: 110.04; standard deviation: 32.72 **b.** ±13.09 **c.** Isabel's claim is reasonable. According to the data, the range of reasonable means is from 96.95 to 123.13, so if Isabel is an average user, she is not exaggerating. **22.** D

Lesson 11-6

2. What Mercedes describes was the alternative hypothesis, rather than the null hypothesis. **4.** H_a: $\mu > 10.8$ **6.** 76.8% to 79.2%; no **8.** Due to the regular rainfall following the grass planting, Becky would be unable to determine whether or not the claims made by the vendor were true. **10.** Since the variable n represents the sample size, the larger the value of n, the larger the denominator will be. Since the denominator represents the divisor, the larger the divisor, the smaller the quotient will be, which in this case is the margin of error. **12.** Null hypothesis: The strength training did not improve the runner's speed in the 100-yd dash, H_0: $\mu \geq 18$. Alternative hypothesis: The strength training improved the player's speed in the 100-yd dash, H_a: $\mu < 18$. **14.** The difference in means in the original groups is –0.4. Here is a sample histogram:



In this case, it is clear that –0.4 is in an area with many other differences generated by the random samples, so you would conclude that you do not have enough evidence to support the alternative hypothesis and that the strength training does not seem to improve times for the 100-yd dash. Since different samples will lead to different results, accept all reasonable attempts.

Selected Answers

Topic 11

16. sample mean without fertilizer: $\overline{x} = 44.8$; sample mean with fertilizer $\overline{x} = 45.0$; The difference in the sample means is $44.8 - 45 = -0.2$. The average yield of bushels of soybeans with the fertilizer is slightly greater than the average yield of the bushels of soybeans without the fertilizer. Since the difference is so small, more testing would definitely be required in order to determine whether the fertilizer was responsible for the higher yield. **18.** H_o: The circumference of the baseball has a mean $\mu = 232$ mm; H_a: The circumference of the baseball has a mean $\mu \neq 232$ mm, where $229 \text{ mm} \leq \mu \leq 235$ mm. **20.** D

Topic Review

2. margin of error **4.** null hypothesis **6.** *z*-score **8.** yes **10.** quantitative **12.** sample survey **14.** This is systematic sampling. The rule is that every 10th person in line was chosen. This is a random, unbiased sample.
16. mean: 76.3; standard deviation: 7.0; minimum: 65; first quartile: 68; median: 77; third quartile: 82; and maximum: 88

18. measure of center: median; measure of spread: interquartile range **20.** When the mean is equal to the median, a data distribution is likely to be normally distributed. **22.** 0.9265, or 92.65% **24.** 0.0174, or 1.74%
26. 0.0606, or 6.06% **28.** Hana found the percent of all values in a normal distribution with $z \leq 1.05$. The percent of all values in a normal distribution with $z \geq 1.05$ is 14.69%. **30.** 17%; ±5%
32. As the sample size increases, the margin of error decreases. In both formulas for determining the margin of error, the value in the denominator of the fraction is $\sqrt{n}$, where n is the sample size. As the denominator of a fraction increases, the value of the fraction decreases. **34.** H_0: $p \leq 0.305$; H_a: $p > 0.305$ **36.** SportORiffic's claim is supported by the study, because the claim is within the range of reasonable means: 20.6 to 25.4.

Selected Answers

Topic 12

Lesson 12-1

2. The probability is greater if the first marble is not returned to the bag. If the first marble is returned, there will be 6 marbles in the bag, of which 4 are red. If it is not returned, there will only be 5 marbles in the bag, of which 4 are red. $\frac{4}{5} > \frac{4}{6}$. **4.** Events are mutually exclusive if they do not share outcomes. Independent events can share outcomes, but the occurrence of one cannot affect the probability of the other happening. **6.** 44% **8.** $\frac{1}{4}$, 0.25, or 25% **10.** The events C and M are not mutually exclusive, so it is not true that $P(C \text{ or } M) = P(C) + P(M)$. So, subtract the probability that a student is in both clubs to find the probability that a random student is in the Chess Club or the Math Club. $P(C \text{ or } M) = P(C) + P(M) - P(C \text{ and } M)$. **12.** yes **14.** no **16.** 0.5, 50%, or $\frac{1}{2}$; The area of the triangle is half of the area of the rectangle because the rectangle and the triangle have the same base (50 cm) and the same height (40 cm). **18.** 0.68 or 68% **20.** 21% **22.** 32% **24. a.** 0.64, or 64% **b.** 0.04, or 4% **26.** C

Lesson 12-2

2. The sample space for $P(B \mid A)$ has to take into account that A is necessary. The sample space for $P(B)$ includes all of B, even the parts that do not include A. **4.** From the formula for conditional probability, both $P(A) \bullet P(B \mid A)$ and $P(B) \bullet P(A \mid B)$ are equal to $P(A \text{ and } B)$. **6.** No; the coach does not know whether girls and boys at the camp are equally likely to play soccer. **8. a.** dependent; $\frac{1}{90}$ **b.** independent; $\frac{1}{100}$ **10.** When A and B are independent, $P(B \mid A) = P(B)$. Then the formula $P(A \text{ and } B) = P(A) \bullet P(B \mid A)$ simplifies to $P(A \text{ and } B) = P(A) \bullet P(B)$, which is true when events A and B are independent. **12.** Without replacement: $P(\text{yellow second} \mid \text{blue first}) = \frac{3}{14}$, which is greater than $P(\text{yellow second} \mid \text{yellow first}) = \frac{2}{14} = \frac{1}{7}$; With replacement: $P(\text{yellow second} \mid \text{blue first}) = P(\text{yellow second} \mid \text{yellow first}) = \frac{3}{15} = \frac{1}{5}$. **14.** 0.6 or 60% **16.** about 0.53 or 53% **18.** Dependent; $P(\text{Game Design} \mid \text{Sophomore}) \approx 53\%$ and $P(\text{Game Design}) = 50\%$, so $P(\text{Game Design} \mid \text{Sophomore}) \neq P(\text{Game Design})$. **20.** 45% **22.** No; $P(\text{improved} \mid \text{medication}) \approx 45\%$ while $P(\text{improved} \mid \text{placebo}) \approx 57\%$. Patients taking the medication showed improvement less frequently than patients taking the placebo. **24.** 0.01 or 1%; Answers may vary. Sample: $P(\text{prize}) = 0.05$ and $P(\text{comic} \mid \text{prize}) = 0.2$, so $P(\text{prize and comic}) = P(\text{prize}) \bullet P(\text{comic} \mid \text{prize}) = (0.05)(0.2) = 0.01$

26. 0.3 or 30%; $P(A \mid \text{defective})$

$$= \frac{P(A \text{ and defective})}{P(\text{defective})} = \frac{P(\text{defective} \mid A) \bullet P(A)}{P(\text{defective})} = \frac{(0.015)(0.2)}{(0.01)} = 0.3$$

28. C

Selected Answers

Topic 12

Lesson 12-3

2. The number of combinations of n items chosen r at a time is found by dividing the number of permutations, ${}_nP_r$, by r! **4.** ${}_9C_2$ and ${}_9C_7$ are equivalent; ${}_9C_2 = \frac{9!}{2!7!}$ and ${}_9C_7 = \frac{9!}{7!2!}$, so ${}_9C_2 = {}_9C_7$. In general, ${}_nC_r = \frac{n!}{r!(n-r)!}$ and ${}_nC_{n-r} = \frac{n!}{(n-r)!r!}$, so ${}_nC_r = {}_nC_{n-r}$.

6. There are ${}_{12}C_4 = 495$ ways to select a committee of 4 people from a group of 12. Only one of these possible committees contains the 4 people newest to the company.
P(4 newest)

$$= \frac{\text{choose 4 of 4 newest} \bullet \text{choose 0 of 8 older}}{\text{choose 4 of 12 total employees}}$$

$= \frac{{}_4C_4 \bullet {}_8C_0}{{}_{12}C_4} = \frac{1}{495}$ **8.** permutation

10. 165 **12.** $\frac{15}{91}$ **14.** $\frac{15}{91}$

16. Problem 12 is the same as problem 15 and problem 13 is the same as problem 14. If no prize is awarded to an athlete, then both prizes are awarded to non-athletes. If no prize is awarded to a non-athlete, then both prizes are awarded to athletes.

18. a. 18,564; the order does not matter, so there are ${}_{18}C_6 = 18{,}564$ groups of 6 erasers. **b.** 56; there are 8 aliens, so there are ${}_8C_3 = 56$ ways to choose 3 aliens. **c.** 6,720; there are 10 flying saucers, so there are ${}_{10}C_3 = 120$ ways to choose 3 flying saucers. Then multiply by the answer to part b.
d. 0.36; the probability is the ratio of the number of favorable outcomes (the answer to part c) to the total number of possible outcomes (the answer to part a). $\frac{6{,}720}{18{,}564} \approx 0.36$

20. 6; any two points form a line and the order is not important. Because ${}_4C_2 = 6$, 6 lines can be determined by four points, where no set of three points is collinear.
22. combination; 330
24. combination; 126
26. permutation; 1,320

28. a. $\frac{{}_4P_3 \bullet {}_5P_0}{{}_9P_3} = \frac{24}{504} = \frac{1}{21}$

b. $\frac{{}_4P_2 \bullet {}_5P_1}{{}_9P_3} = \frac{60}{504} = \frac{5}{42}$

30. $\frac{9}{28}$; use combinations because which \$1 bill and which \$10 bill do not matter, and the order in which they are pulled out does not matter. $\frac{{}_3C_1 \bullet {}_3C_1}{{}_8C_2} = \frac{9}{28}$

32. a. $\frac{1}{30{,}240}$; order matters, so there are ${}_{10}P_5$ possible codes, only one of which is 30429. So $P(30429) = \frac{1}{{}_{10}P_5} = \frac{1}{30{,}240}$.

b. $\frac{1}{100{,}000}$; there are 10^5 possible codes, only one of which is 30429.
So $P(30429) = \frac{1}{10^5} = \frac{1}{100{,}000}$.

34. no; yes; yes; no; no; no; yes; yes

36. Part A

$P(\text{V1 and V2}) = P(\text{V1}) \bullet P(\text{V2} \mid \text{V1})$

$= \frac{3}{9} \bullet \frac{2}{8} = \frac{1}{12}$

$P(\text{V1 and V2}) = \frac{{}_3C_2}{{}_9C_2} = \frac{1}{12}$

Part B $P(\text{SURF}) = \frac{1}{9} \bullet \frac{1}{8} \bullet \frac{2}{7} \bullet \frac{1}{6} = \frac{1}{1{,}512}$

$P(\text{SURF}) = \frac{{}_1P_1 \bullet {}_1P_1 \bullet {}_2P_1 \bullet {}_1P_1}{{}_9P_4}$

$= \frac{2}{3{,}024} = \frac{1}{1{,}512}$

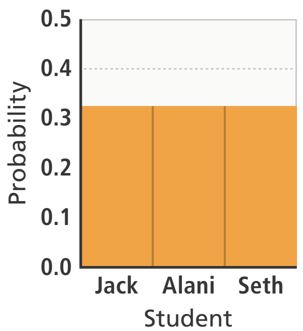
Selected Answers

Topic 12

Lesson 12-4

2. A binomial experiment has specific rules. It can only have two outcomes, the trials must be independent, and the probability for success is the same every trial.

4.



6. 35% **8.** 48% **10.** 90%
12. The probability distribution is the function P, defined on the set {4, 5, 6}, such that $P(4) = 0.25$, $P(5) = 0.5$, and $P(6) = 0.25$. **14.** Abby forgot to multiply by ${}_7C_5$.
$P(5) = {}_7C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2 = 21\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2$
≈ 0.0384.
16. Let P be the function defined on the set {Y, R, G, B} such that $P(\text{Y}) = 0.3$, $P(\text{R}) = 0.4$, $P(\text{G}) = 0.2$, and $P(\text{B}) = 0.1$.
18. Let P be the function defined on the set {0, 1, 2, 3, 4} such that $P(0) = \frac{1}{10}$, $P(1) = \frac{7}{30}$, $P(2) = \frac{3}{10}$, $P(3) = \frac{1}{5}$, and $P(4) = \frac{1}{6}$. **20.** Yes; there are 50 trials; each trial has two possible outcomes, success and not success; the performance of one bulb does not affect the performance of another, so the trials are independent; and the probability of success, 0.9, is the same for each trial. **22.** No; the probability of making the free throws is not the same for each trial. **24.** 18.8%
26. 52.6% **28. a.** 56% **b.** Answers may vary. Sample: no; a sample set of 8 people is too small to make a decision from. In order to make an informed decision, the pharmaceutical company should allow more people to test the new medication. **30.** A, B, C, E
32. Part A The theoretical probability distribution is the function P defined on the set {0, 1, 2, 3, 4, 5} such that $P(0) = 0.03125$, $P(1) = 0.15625$, $P(2) = 0.3125$, $P(3) = 0.3125$, $P(4) = 0.15625$, $P(5) = 0.03125$. (Students may express the probabilities as percents or fractions and they may round decimals/percents.)
Part B Answers may vary. Sample: The experimental probability distribution is the function P defined on the set {0, 1, 2, 3, 4, 5} such that $P(0) = 0.05$, $P(1) = 0.1$, $P(2) = 0.3$, $P(3) = 0.35$, $P(4) = 0.2$, $P(5) = 0$.
Part C The experimental probability distribution is somewhat similar to the theoretical distribution but not identical. A theoretical probability distribution describes the results you would expect if an experiment is repeated many times, and the experiment was only repeated 20 times.

Lesson 12-5

2. He found the probability of heads, not the expected number of heads. The expected number of heads is 50% • 10, or 5. **4.** The class pays an average of \$1.48 in winnings for every lottery ticket sold. **6.** 3.5 **8.** 12
10. \$16.90

Selected Answers

Topic 12

12. Nonrefundable; there is a 20% chance the man will not fly, so there is an 80% chance he will fly. If he purchases the nonrefundable ticket, his cost is $600 whether he flies or not. If he purchases the refundable ticket, his expected cost is (0.8)(900) + 0.2(0) = $720.
Based on expected value, he should purchase the nonrefundable ticket.
14. Sample: A mean is an average of known values. Expected value is the mean of values that are unknown but that follow a known or estimated probability distribution.
16. $600 **18.** Option C; the cost for the car owner for Option A is $900: for Option B, $820; for Option C, $750.
20. 27 days **22.** Since 3 of the 4 tosses were Heads, the student concludes that *P*(heads) is 3 out of 4, or 75%. This is not good reasoning because the sample size is so small. Since a fair coin would have a probability of 50% for Heads, the student should draw a much larger sample before concluding that the probability is not 50%. **24.** $60,268,000
26. C

Lesson 12-6

2. Generate numbers 1 to 6 on a calculator or using index cards, where each number represents the same number on a number cube. **4.** A fair game requires that participants have an equal chance to win or an expected value of 0, meaning no participant has an advantage. **6.** If a "win" is represented by a positive 1 and a "loss" is represented by a negative 1 and there is an equal chance that a player will win or lose, the sum of +1 and –1 is 0.

8. (1) Write each friend's name on a slip of paper, and draw one slip at random from a paper bag to determine who gets the last slice. Each slip has a $\frac{1}{3}$ chance of being chosen. (2) Assign the numbers 1 and 2 to one friend, 3 and 4 to a second friend, and 5 and 6 to the third friend. Roll a number cube, and give the slice to the friend whose number lands on top. Each person has a $\frac{2}{6}$ chance of being chosen.
(3) Assign each friend a number from 0 to 2. Use a calculator to generate a random number from 1 to 9. Divide the number by 3, and give the slice to the friend whose number matches the remainder. Each person has a $\frac{3}{9}$ chance of being chosen. **10.** 0.659, 65.9%
12. No; there are 2 ways to get 1 head since the sample space is {HH, HT, TH, TT}.
14. Assign numbers 0 through 7 to each of the candidates, and then generate one of the eight integers randomly three times in a row. If the same number (student) is selected during the second or third trials, ignore the number and generate another number. **16.** Unfair; even numbers are far more likely (27 out of 36 possibilities) than odd numbers.
18. She should keep what she has (and not spin) because the game is not fair and she is more likely to lose money than win. **20. a.** $19.71 **b.** $96.00,
c. Since 12 mo × $5.49/mo = $65.88, the cost of the insurance for a year is $65.88. Since the repair costs could be as much as $1,200 and the probability of a leak could be 8%, the expected costs of a gas leak without insurance could be as high as $96.00. So I would advise her to buy the insurance.

Selected Answers

Topic 12

22. A, B, D **24. Part A** Model 1001: \$48; Model 1002: \$50; Calculate the expected profit after potential lawsuits for each model like this: Model 1001:

$$\$60 - (\$1{,}200{,}000)\left(\frac{2}{200{,}000}\right) = \$48$$

Model 1002:

$$\$56 - (\$1{,}200{,}000)\left(\frac{1}{200{,}000}\right) = \$50$$

Part B It is recommended that the company stop selling Model 1001 and only sell Model 1002. The company can expect to make more money on Model 1002 than on Model 1001. Also, Model 1002 is a safer tire; fewer people will be injured or die if they have Model 1002 tires than if they have Model 1001 tires.

Topic Review

2. permutation **4.** dependent events **6.** expected value **8.** mutually exclusive **10. a.** 0; The probability he will roll a number that is both even and less than 2 is 0 because these are mutually exclusive events. It is impossible to roll a number that lies in both sets. **b.** $0.\overline{6}$ or $66.\overline{6}\%$; Because the events are mutually exclusive, you can add the probabilities.

$$P(<2) + P(\text{even}) = \frac{1}{6} + \frac{1}{2} = \frac{2}{3} = 0.\overline{6} = 66.\overline{6}\%$$

12. 0.30 **14.** approx. 0.21
16. The student's conclusion is correct even though the reasoning is wrong. To test for independence, the student should have compared $P(\text{prime} \mid \text{even})$ with $P(\text{prime})$, or $P(\text{even} \mid \text{prime})$ with $P(\text{even})$. $P(\text{prime} \mid \text{even}) = 0.5$ and $P(\text{prime}) = 0.6$, so the events are dependent. **18.** permutation; 56
20. The student computed ${}_5P_2$ instead of ${}_5C_2$. The student needs to divide by 2! to complete the calculation. ${}_5C_2 = 10$. **22.** 0.35% **24.** 22.89%
26. Akasi mixed up the variables *n* and *r*. By putting them in the correct place, she can get her solution.

$$P(3) = {}_5C_3 \bullet 0.24^3(1 - 0.24)^{5-3}$$
$$\approx 0.0798 \approx 7.98\%$$

28. 345