Selected Answers

Topic 1

Lesson 1-1

1. The domain and range of a function give information about all the possible inputs and outputs for the relationship. The *x*- and *y*-intercepts of the function's graph give information about what happens to the relationship when one of the quantities is 0. The graph also shows whether the quantities in the relationship are positive or negative, whether one is increasing or decreasing with respect to the other, and what the rate of that increase or decrease is.
3. Lonzell confused positive with increasing and negative with decreasing. The function is positive on the intervals $(-\infty, -4)$ and $(2, \infty)$ and negative on the interval $(-4, 2)$.
5. $[-1, 3]$ **7.** $y = 1$ **9.** $(-4, -2)$ and $(2, 4)$ **11.** 0 **13.** The zeros, or values along the x-axis, are neither positive nor negative. These values should not be included in the interval over which the function is positive. The function is positive on the interval $(-1, 3)$ and negative on the intervals $(-\infty, -1)$ and $(3, \infty)$. **15.** No; linear functions can also be constant, which means neither increasing nor decreasing.
17. When the graph is increasing, the speed is increasing. When the graph is decreasing, the speed is decreasing.
19. *x*-intercepts: −4, 2; *y*-intercept: −8
21. decreasing: $(-\infty, -1)$; increasing: $(-1, \infty)$ **23.** domain: $[-5, 5]$; range: $[-1, 2]$ **25.** positive: $(-5, 3)$; negative: $(3, 5]$ **27.** $-\frac{1}{2}$
29. a. The *x*-intercepts are approximately 0.05, 0.7, 0.8, 1.45, 1.55, and 2.2. The *x*-intercepts represent the moment the jumper is at the height of the frame of the trampoline. The *y*-intercept is −0.5 and represents the sag of the trampoline when the jumper pushes down at the start of the first jump. **b.** (0.05, 0.7), (0.8, 1.45), and (1.55, 2.2); The positive intervals represent the times when the jumper is above the frame of the trampoline.
c. (0, 0.05), (0.7, 0.8), (1.45, 1.55), and (2.2, 2.25); The negative intervals represent the times when the jumper is below the frame of the trampoline.
d. 6 feet per second; The average rate of change over the interval [0.75, 1.125] represents the average speed of the jumper on the upward part of a jump.
31. a. No **b.** Yes **c.** No **d.** Yes
33. Part A −2,000 gallons per hour; Over the interval [0, 4], the water in the cistern is being used at a rate of 2,000 gallons per hour. 1,500 gallons per hour; Over the interval [6, 10], the cistern is being filled with water at a rate of 1,500 gallons per hour.
Part B The water in the cistern is not being used, and the cistern is not being filled with water. Therefore, the amount of water in the cistern is constant. **Part C** −200 gallons per hour; From 0 hours to 10 hours, the water in the cistern is being used at a rate of 200 gallons per hour. No, this rate of change shows the change in the starting amount of water at 0 hours and the ending amount of water at 10 hours, but it does not indicate that the amount of water decreased from 0 hours to 4 hours, remained the same from 4 hours to 6 hours, and increased from 6 hours to 10 hours.

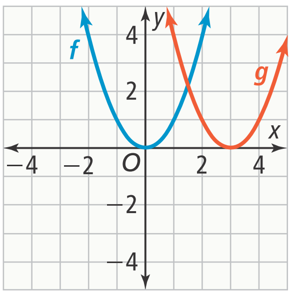
Selected Answers

Topic 1

Lesson 1-2

1. From the equations, you can see whether the graph of the parent function will be shifted up or down, reflected over an axis, or stretched or compressed to create the graph of the new function. **3.** $g(x)$ is a horizontal translation of $f(x)$ left 1 unit, not up 1 unit, because $h < 0$.

5.



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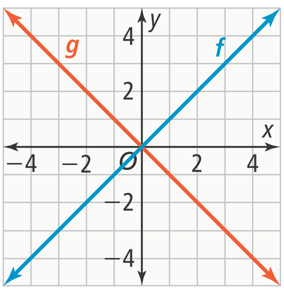

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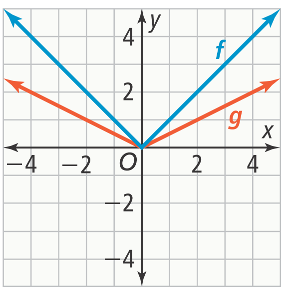

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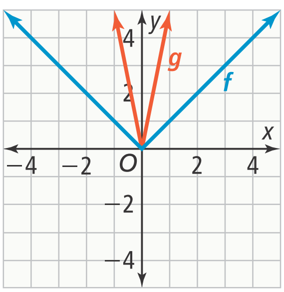
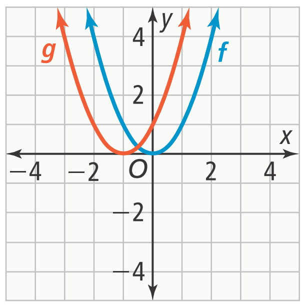


13. $g(x) = (x - 3)^2 - 6$ **15.** Multiplying the y-values of the points on the graph of f by 4 results in points on the graph of g. So, $g(x)$ is a vertical stretch of $f(x)$ by a scale factor of 4 and can be written as $g(x) = 4|x|$. Dividing the x-values of the points on the graph of f also results in points on the graph of g. So $g(x)$ is a horizontal compression of $f(x)$ by a scale factor of 4 and can be written as $g(x) = |4x|$. A vertical stretch is the same as a horizontal compression.

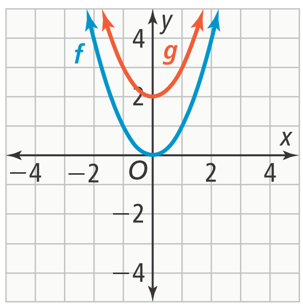
17. $g(x) = |2x - 4|$ is equivalent to $g(x) = 2|x - 2|$. So, $g(x)$ is a horizontal translation of $f(x)$ right 2 units and a vertical stretch of $f(x)$ by a factor of 2.

19.



The range values of f and g are the same, but the domain values of g are 1 unit less than the domain values of f at corresponding range values.

21.

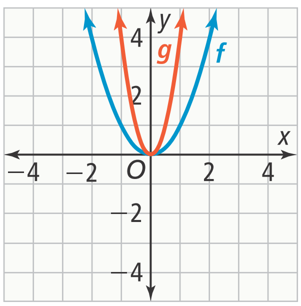


The domain values of f and g are the same, but the range values of g are 2 units greater than the range values of f. **23.** $f(x) = x^2 + 1$

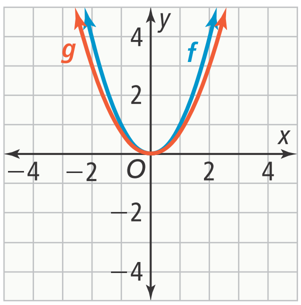
Selected Answers

Topic 1

25.



27.

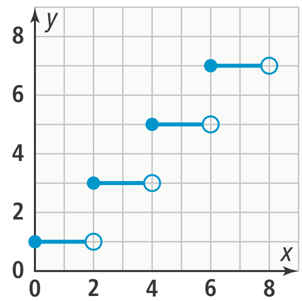


29. translation left 1 unit and then vertical stretch by a factor of 2
31. reflection across the x-axis and then translation down 6 units
33. $y = (x - 3)^2 + 9$
35. $y = -(x - 1)^2 + 1$; $(1, 1)$ **37.** A

Lesson 1-3

1. Model each part of its domain separately, and then combine them into one function using a brace, including information that details what models goes with that part of the domain. **3.** There is an open circle on the piece of the function for $y = -4x - 7$, but there is a closed circle on the piece of the function for $y = 2x + 5$. **5.** No; Sample: Over the interval $[-2, 2)$ for x, each input to the rule gives two outputs, so this is not a function.

7.



9. $f(x) = \begin{cases} 3, \text{ if } -2 < x \leq 0 \\ \frac{1}{2}, \text{ if } \ 0 < x \leq 2 \\ -2, \text{ if } \ 2 < x \leq 4 \end{cases}$

11. $\{x \mid x \neq 3\}$ **13.** Answers may vary. Samples: $x = 2$ and $x = 8$

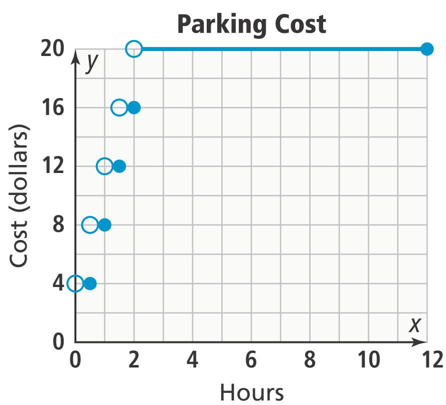
15.

$C(x) = \begin{cases} 25, & \text{when } \ 0 < x \leq 250 \\ 0.20x + 25, & \text{when } \ x > 250 \end{cases}$

17. $y = \begin{cases} x + 5, & \text{when } \ x < -1 \\ -2, & \text{when } \ x > 1 \end{cases}$

19. $f(x) = \begin{cases} -2x - 6, & \text{if } \ x < -3 \\ 2x + 6, & \text{if } \ x \geq -3 \end{cases}$

21.



Selected Answers

Topic 1

23. a. no message plan: $f(x) = 0.10x$; 500 message plan:

$$f(x) = \begin{cases} 7.99, \text{ when } 0 \le x \le 500 \\ 0.10x - 42.01, \text{ when } x > 500 \end{cases}$$

1,200 message plan:

$$f(x) = \begin{cases} 13.99, \text{ when } 0 \le x\ 1200 \\ 0.10x - 106.01, \text{ when } x > 1200 \end{cases}$$

unlimited message plan: $f(x) = 24.99$
b. no message plan: \$150; 500 message plan: \$107.99; 1,200 message plan: \$43.99; unlimited message plan: \$24.99 **c.** no message plan: $x < 80$; 500 message plan: $80 \le x \le 560$; 1,200 message plan: $560 \le x \le 1310$; unlimited message plan: $x \ge 1310$

25. a. yes **b.** no **c.** yes **d.** no

27. $y = \begin{cases} 20x, & 0 \le x \le 40 \\ 30x - 400, & 40 \le x \le 70 \\ 40x - 1{,}100, & 70 \le x \le 100 \end{cases}$

Lesson 1-4

1. If a problem has a set of numbers with a common difference, you can create and use an explicit formula in order to solve the question. **3.** The pattern is + 1, + 2, + 3, … . While this is a pattern, there is no common difference between consecutive terms. This is not an arithmetic sequence.
5. $\frac{1}{4}, \frac{3}{2}, \frac{7}{4}, 2$ **7.** 12; 263, 275, 287 **9.** 2.2; 12.9, 15.1, 17.3 **11.** \$294 **13.** The common difference is the constant change between consecutive terms in an arithmetic sequence. Sample: In the arithmetic sequence 2, 4, 6, 8, …, the common difference is a positive 2.

15. Both; Gregory's formula can be simplified to get Felipe's formula.
17. a. $a_n = 2 + 0.3(n - 1)$ **b.** day 38
19. yes; –11; 53 **21.** yes; 4; 19
23. $a_n = \begin{cases} -2, \text{ if } n = 1 \\ a_{n-1} + 7, \text{ if } n > 1; \end{cases}$ $a_n = 2 + 7(n - 1)$
25. $a_n = \begin{cases} -4, \text{ if } n = 1 \\ a_{n-1} - 4, \text{ if } n > 1; \end{cases}$ $a_n = -4 - 4(n - 1)$
27. 175 **29.** 165 **31.** 3,815 seats
33. 6409 ft **35. a.** $a_n = 18 + 3(n - 1)$
b. $a_1 = 18$; $a_n = a_{n-1} + 3$ **c.** 63 push-ups
d. 648 push-ups **37.** B

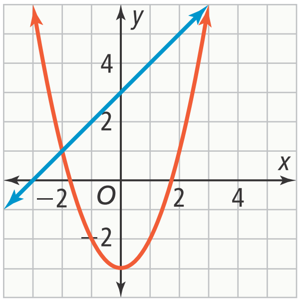
Lesson 1-5

1. To solve an equation by graphing, write two new equations by setting y equal to the expression on either side of the equals sign. Then graph the two equations and identify the points of intersection. **3.** No; Ben found the solution of the inequality $-x^2 + 9 < 0$. The correct solution is $-3 < x < 3$.
5. no solution; Subtracting $3x$ from both sides of the original equation gives $-5 = 2$, which is false, so there is no solution. **7.** $2x + 3 = -|x + 1| - 1$; The graph shows the solution is at $x = -3$. Substitute –3 for x in the equation: $2(-3) + 3 = -|(-3) + 1| - 1$. Simplify the equation to see if it results in a true statement: $-3 = -3$. **9.** $x = 2$
11. $x = 1$ **13.** $x = 8$ **15.** $x < -1$ or $x > 8$
17. $-3 < x < 2$ **19.** $x \le -5$ or $x \ge 3$
21. $4x > 2x + 6$; Tamira will pass Cindy after 3 hours. **23.** $x \approx -0.554$ and $x \approx 1.804$ **25.** $x \approx 1.857$ **27.** $x \approx 0.3$ and $x \approx 7.7$ **29.** $x \approx -4.3$ and $x \approx 1.3$
31. a. $-16t^2 + 24t = 3$ **b.** about 1.36 s

Selected Answers

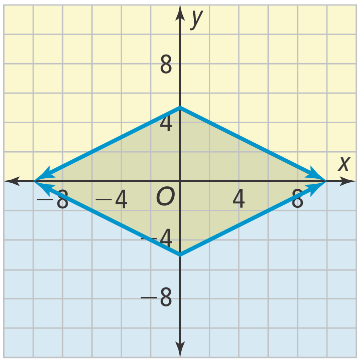
Topic 1

33.



$x = -2$ and $x = 3$

35. Part A

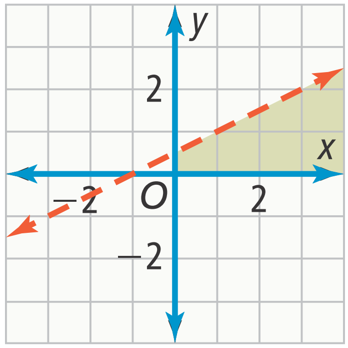


Part B parallelogram **Part C** 50 ft^3

Lesson 1-6

1. Graphs of the equations can be shown on a coordinate plane. The point of intersection of the lines can show the solution of the system. Graphs of the inequalities can also be shown on a coordinate plane. The overlapping regions of the shaded sections can show the solution of the system.
3. A system of linear inequalities has to be solved graphically to see the regions of overlap. **5.** A coefficient matrix only includes the coefficients of the variables in a system of equations. However, an augmented matrix also includes the constant terms.

7.

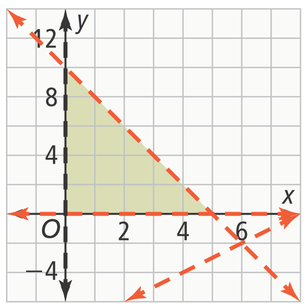


9. Two lines that intersect at one point have one solution. Two lines that never intersect have no solutions. Two lines that always intersect have infinitely many solutions. **11.** –24 in the second equation was not multiplied by 2. –24 should be –48. The solution is (–6, 3).
13. Answers may vary. Sample:

$$\begin{cases} x + y + z = 6 \\ x + y - z = 0; \ (1, 2, 3); \\ x - y + z = 2 \end{cases}$$

Start with the solution. Then write 3 equations that are true for the solution.
15. (–1, 2); $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$ **17.** (2, 7)
19. (–2, –1)

21.



23. (–2, 3, – 1) **25.** $\left[\begin{array}{cc|c} -2 & 1 & 0 \\ 4 & -1 & 9 \end{array}\right]$

27. $\left[\begin{array}{cc|c} 1 & -7 & -1 \\ -1 & -1 & -1 \end{array}\right]$ **29.** $\begin{cases} 0.5x + y = 0 \\ -x + 4y = 2 \end{cases}$

Selected Answers

Topic 1

31. Let x represent the number of triangular tiles and y represent the number of square tiles.

$\begin{cases} x + y = 50 \\ 3x + 4y = 170 \end{cases}$; $\left[\begin{array}{cc|c} 1 & 1 & 50 \\ 3 & 4 & 170 \end{array}\right]$

33. more than 10 memberships per week **35.** 2 mL of the 50% saline solution and 8 mL of the 25% saline solution **37.** B

Lesson 1-7

1. A system of equations can be written as a matrix, and then row operations can be applied so that the matrix transforms into reduced row echelon form. This form reveals the solution of the system of equations. **3.** The row at the bottom did not need to be altered any further—a row of all zeros at the bottom can be part of an rref matrix.

5. $\left(3, \frac{1}{2}\right)$ **7.** $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 8 \end{array}\right]$ **9.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

11. Yes; write the equation of each line. Then write a matrix to represent the system. Write the matrix in reduced row echelon form. The last column gives the point of intersection of the $f(x)$ and $g(x)$. **13.** Multiplying or dividing a row in a matrix is related to writing an equivalent equation.
15. When a system has three equations in three variables, technology should be used to find a matrix in reduced row echelon form. **17.** (5, –3) **19.** (2, –2, 3)

21. $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 8 \end{array}\right]$ **23.** $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -6 \end{array}\right]$

25. infinitely many solutions

27. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 0 & 1 & -2 & 5 \\ 2 & -1 & 0 & -15 \end{array}\right]$, $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 95 \\ 0 & 0 & 1 & 45 \end{array}\right]$;

$A = 40°$; $B = 95°$; $C = 45°$

29. \$19 **31.**

1	0	0	2
0	1	0	–3
0	0	1	1

33. Part A $\begin{cases} y = 2x \\ 2x + y = 30 \end{cases}$

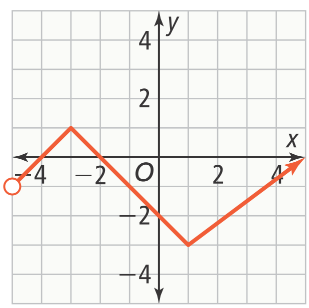
Part B $\left[\begin{array}{cc|c} -2 & 1 & 0 \\ 2 & 1 & 30 \end{array}\right]$ **Part C** $\left[\begin{array}{cc|c} 1 & 0 & 7.5 \\ 0 & 1 & 15 \end{array}\right]$

Part D The length of side x is 7.5 cm and the length of side y is 15 cm.

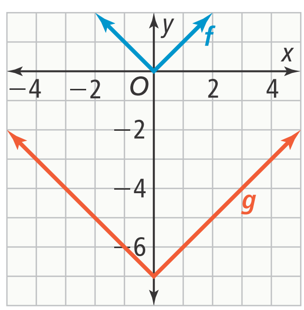
Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** maximum
5. zero of the function **7.** domain: (–5, ∞); range: [–4, 1]; zeros: $x = 1$ and $x = 3$; positive: (1, 3); negative: (–5, 1) and (3, ∞)

9.



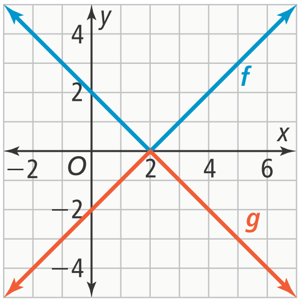
11.



Selected Answers

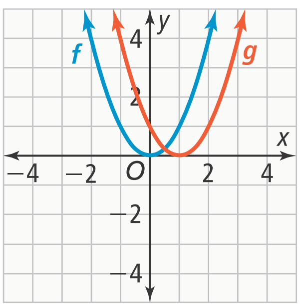
Topic 1

13.

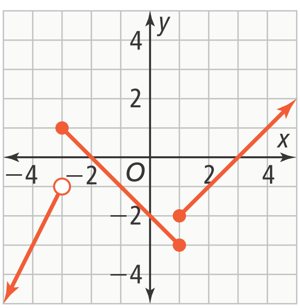


15. Let $f(x) = |x|$. When $k > 0$, $g(x) = kf(x)$ represents a vertical stretch of the absolute value function. For every x-value, each y-value of g is k times farther from the x-axis than the corresponding y-value for f. The function $h(x) = \left|\frac{1}{4}x\right|$ produces a horizontal stretch by a factor of k. For every y-value, each x-value of h is k times farther from the y-axis than the corresponding x-value for f.

17.



19.

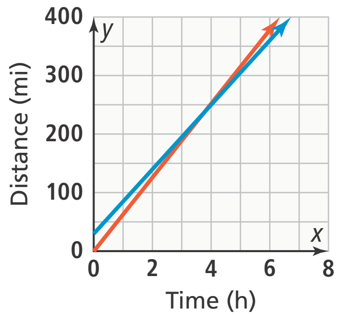


21. Yes; The graph of every absolute value function is composed of two branches, one of which is increasing and the other decreasing. **23.** $d = 12$; 51

25. $a_n = \begin{cases} 5, \text{ if } n = 1 \\ a_{n-1} + 4, \text{ if } n > 1; \end{cases}$

$a_n = 5 + 4(n - 1)$

27. 304 **29.** 144 **31.** 81 cubes
33. $x = -11$ and $x = 5$ **35.** $(-1, 8)$
37. The car is ahead of the truck when $63x > 55x + 30$, or after 3.75 h.



39. $x = 2$, $y = -2$

41. $\begin{cases} 12x + 20y \le 320 \\ x \ge 5 \\ y \ge 8 \end{cases}$

5 tables and 13 chairs or 7 tables and 10 chairs

43. $5x + 2y = 6$
$6x - 7y = -4$

45. red: 2 points; white: 5 points; blue: 10 points

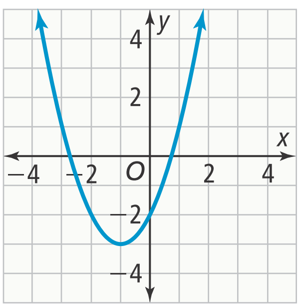
Selected Answers

Topic 2

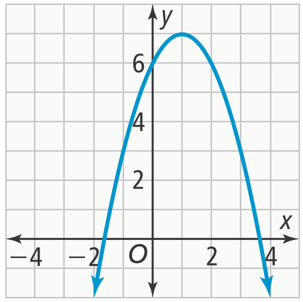
Lesson 2-1

1. From the vertex form $f(x) = a(x - h)^2 + k$, you can find the vertex (h, k) and see whether the parabola opens upward (if $a > 0$) or downward (if $a < 0$). From a graph, you can read the coordinates of the vertex and substitute into the vertex form. With the coordinates of one other point, you can solve the equation to find a and write the equation of the quadratic function in vertex form.
3. A quadratic function has the shape of a parabola. **5.** The graph of the function g is a reflection across the x-axis and a translation 5 units to the left and 2 units up. **7.** $y = -12(x + 3)^2 + 7$
9. $y = -2(x + 4)^2 + 6$
11. $g(x) = (x + 1)^2 + 3$
13. $y = -2(x + 3)^2 + 4$
15. a. The ball is 18 ft away from Amaya when it is at its maximum height. Use the x-coordinate of the vertex. **b.** Substitute 30 for the variable x in the equation. The ball's height is 9.12 ft.

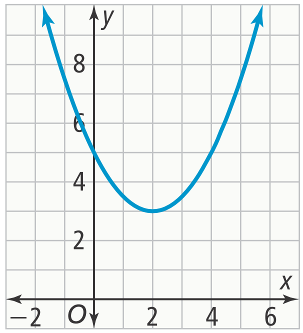
17. The graph of the function is translated 1 unit left and 3 units down from the graph of $f(x) = x^2$.



19. The graph of the function is a reflection in the x-axis, translated 1 unit right and 7 units up from the graph of $f(x) = x^2$.



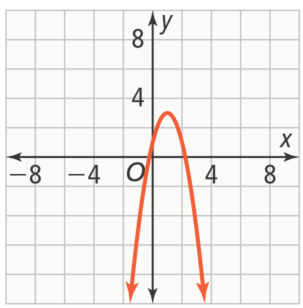
21. The graph of the function is a translation 2 units right and 3 units up with a vertical compression of the graph of $f(x) = x^2$.



23. Vertex: (1, 2); axis of symmetry: $x = 1$; maximum: $y = 2$; domain: $(-\infty, \infty)$; range: $(-\infty, 2]$ **25.** Vertex: $(-2, -1)$; axis of symmetry: $x = -2$; minimum: $y = -1$; domain: $(-\infty, \infty)$; range: $[-1, \infty)$ **27.** $y = -\frac{4}{9}(x - 3)^2 + 6$
29. $y = 2(x + 1)^2 - 6$; $y = 2x^2 + 4x - 4$
31. $g(x) = -(x - 3)^2$ **33.** Answers will vary: Sample: $(2, -5)$, $(3, -14)$, $(-1, -14)$
35. $y = -(x - 2)^2 + 9$ **37.** B

Lesson 2-2

1. axis of symmetry, vertex point, and y-intercept **3.** any quadratic function in standard form; Sample: $y = 2x^2 + 6x - 1$
5. vertex: (2, 28); y-intercept: (0, 40)
7. Maximum: 52 **9.** $y = -x^2 - 6x + 6$
11.



Selected Answers

Topic 2

13. Find the y-coordinate of the vertex. **15.** Micah forgot to use the sign associated with the value of b.

$x = -\frac{b}{2a}$ $\quad x = -\frac{-47.5}{2(-9.5)}$ $\quad x = \frac{47.5}{-19}$

$x = -2.5$

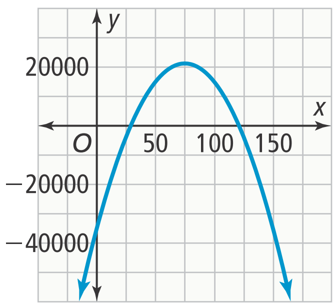
$y = -9.5(-2.5)^2 - 47.5(-2.5) + 63$

$= -59.375 + 118.75 + 63$

$= 122.375$

17. $(-2, -17)$ **19.** vertex: $(4, -5)$; y-intercept: $(0, 11)$ **21.** vertex: $(-3, 13)$; y-intercept: $(0, -5)$ **23.** $y = -x^2 + 2x + 8$ **25.** $y = -16x^2 + 10x + 35$

27. a.



b. 75 admission counselors **c.** \$21,250 **29.** A, C, D **31. Part A** $y = -5.0x^2 + 10.2x + 24.9$ **Part B.** 24.9 ft; 30.1 ft **Part C.** about 2.75 seconds; Diving from the halfway point to the water is faster.

Lesson 2-3

1. The factored form is helpful because you can use it and the Zero Product Property to find the zeros of quadratic equations. **3.** When a quadratic equation is written in factored form, each factor can be set equal to zero to find the zeros, because of the Zero Product Property. **5.** $(x - 8)(x + 3)$ **7.** $x = 2$, $x = 10$ **9.** 5 seconds **11.** The student misapplied the Zero Product Property. The student should have factored $2x(x + 1) + 5(x + 1)$ as $(2x + 5)(x + 1)$ first, and then applied the Zero Product Property. Then $2x + 5 = 0$ leads to $x = -\frac{5}{2}$, and $x + 1 = 0$ leads to $x = -1$.
13. for all values of x except 4
15. the factored expression $(x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 25 = x^2 + 10x + 25$; The expression $x^2 + 25$ does not factor over the real numbers.
17. $(x - 5)(x + 2)$ **19.** $(x + 8)(x + 7)$
21. $3(x + 2)(x - 8)$ **23.** 5, −4 **25.** 2, 3
27. $-3, \frac{3}{5}$ **29.** $-\frac{2}{3}, \frac{3}{2}$ **31.** $x < -6$, $x > -3$
33. $x < -3$, $x > 8$ **35.** $x \neq -3$
37. $y = 2x^2 - 4x - 6$ or $y = 2(x + 1)(x - 3)$
39. $y = \left(\frac{11}{27}\right)(x - 1)(x - 11)$
41. a. $y = 0.1(x - 5)(x - 10)$
b. 5 seconds and 10 seconds; The drone's height will be 0, so use the factored form of the equation and the Zero Product Property to find that $x - 5 = 0$ and $x - 10 = 0$, or $x = 5$ and $x = 10$. **c.** 15 seconds **43.** B, C
45. Part A $y = -16x^2 + 72x$; The zeros of the function are at 0 and 4.5, and the peak is halfway between, since it is a quadratic function, so the point (2.25, 81) represents its peak;
$y = ax(x - 4.5)$; $81 = 2.25a(2.25 - 4.5)$;
$81 = -5.0625a$; $-16 = a$;
$y = -16x(x - 4.5)$; $y = -16x^2 + 72x$
Part B The pumpkin will reach 100 feet at 2.5 seconds.

Selected Answers

Topic 2

Lesson 2-4

1. Numbers that are not on the real number line are not real numbers. However, by defining the *imaginary unit* $i = \sqrt{-1}$, we can work with a larger set of numbers. We call these *complex numbers.* They are written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. Complex numbers represented in this way can be added, subtracted, multiplied, divided, and otherwise operated on even though they are not necessarily real numbers on the real number line.
3. She correctly multiplied the quotient by one, but she used $\frac{3-i}{3-i}$ where she should have used $\frac{3+i}{3+i}$. The idea is to use the complex conjugate of the denominator. **5.** $8 + 4i$ **7.** $4i, -4i$
9. $2 - 4i$ V **11.** The student has multiplied both the numerator and denominator by their complex conjugates, which is an invalid operation. It changes the quotient rather than just reducing it to a different form. He or she should have multiplied the quotient by the complex conjugate of the denominator over itself. Since this is equivalent to multiplying the quotient by one, it is a valid operation:

$$\frac{1+i}{3-i} = \frac{1+i}{3-i}\left(\frac{3+i}{3+i}\right) = \frac{3+4i+i^2}{9-i^2} = \frac{1}{5} + \frac{2}{5}i$$

13.
$$(a+bi) \div (c+di) = \frac{a+bi}{c+di} = \frac{a+bi}{c+di}\left(\frac{c-di}{c-di}\right) = \frac{ac - adi + cbi + bd}{c^2+d^2} = \frac{(ac+bd)+(cb-ad)i}{c^2+d^2}$$

15. $\pm 0.1i$ **17.** 1 **19.** $-1 + 2i$ **21.** $9 - 6i$
23. $-2 + 8i$ **25.** $12 + 15i$ **27.** $55 + 48i$
29. 53 **31.** $\frac{3}{4} - \frac{1}{4}i$ **33.** $-\frac{4}{5} - \frac{8}{5}i$
35. $(x + i)(x - i)$ **37.** $2(3y - 2i)(3y + 2i)$
39. $(x + yi)(x - yi)$ **41.** $\frac{3}{5}i, -\frac{3}{5}i$ **43.** $\frac{2}{7}i, -\frac{2}{7}i$
45. $\frac{1}{2}i, -\frac{1}{2}i$ **47. a.** $3 + 2i$ **b.** $i(3 + 2i) = 3i + 2i^2 = 3i - 2 = -2 + 3i$. $-2 + 3i$ can be interpreted as the point $(-2, 3)$.
c. Multiplication by i reverses the coordinates of the original point and then multiplies the x-coordinate by -1. In other words, the point (a, b) always becomes $(-b, a)$. This is a rotation of 90° counterclockwise. **49.** B

Lesson 2-5

1. Completing the square is used to rewrite a quadratic equation to make it easier to solve. Once the quadratic is a perfect square form, you can solve it by taking the square root of each side of the equation and then isolating the variable. **3.** Find half of b, then square that result. **5.** $-1, -11$
7. $\frac{(-3 + \sqrt{37})}{2}, \frac{(-3 - \sqrt{37})}{2}$
9. $y = -2(x - 5)^2 + 8$; maximum $(5, 8)$
11. You can compare the zeros of the graph to the solutions that you calculated and, because completing the square rearranges the equation to vertex form, you can also compare the vertices.

Selected Answers

Topic 2

13. $\frac{49}{16}$; Half of $\frac{7}{2}$ is $\frac{7}{4}$, and $\left(\frac{7}{4}\right)^2 = \frac{49}{16}$. **15.** The student forgot to divide the *b*-term by 2 before squaring it. He or she should have added 16 to each side of the equation to complete the square. **17.** Answers may vary. Sample: The vertex form of the equation is more helpful because you can immediately identify the vertex of the graph. **19.** 1, 9 **21.** $6 \pm \sqrt{\frac{5}{3}}$
23. $-7 \pm 2i$ **25.** $(x + 11)^2 = 0.5$
27. $\left(x + \frac{1}{2}\right)^2 = \frac{15}{4}$ **29.** $(x + 0.3)^2 = 19.19$
31. $-17, 3$ **33.** $-4 \pm \sqrt{\frac{134}{7}}$
35. $0.2 \pm \sqrt{1.24}$ **37.** $-\frac{7}{2}, \frac{3}{2}$ **39.** $-3 \pm 3\sqrt{2}$
41. $-9 \pm 7\sqrt{2}$ **43.** $y = (x + 2)^2 - 17$; minimum $(-2, -17)$
45. $y = -2(x + 5)^2 - 8$; maximum $(-5, -8)$ **47.** $y = 6(x - 3.5)^2 + 1$; minimum $(3.5, 1)$ **49.** 5.98 seconds; 254 feet
51. 2002 **53.** C

Lesson 2-6

1. The Quadratic Formula provides the solutions to a quadratic equation in the form $ax^2 + bx + c = 0$. The discriminant in the formula, $b^2 - 4ac$, lets you predict the number and type of solutions. If $b^2 - 4ac > 0$, there will be 2 real solutions. If $b^2 - 4ac = 0$, the equation has 1 real solution. If $b^2 - 4ac < 0$, then the equation has 2 nonreal solutions. **3.** Rick may be looking at the graph for real solutions. The discriminant for this equation is $5^2 - 4(1)(9) = 25 - 36 = -11$. There are no real-number solutions for this equation, but there are 2 nonreal solutions.
5. 2 non-real solutions
7. 6.7 seconds **9.** Sample: The Quadratic Formula provides the solutions of the equation. If *h* and *k* are solutions of a quadratic equation, then the quadratic expression can be factored as $(x - h)(x - k)$. **11.** The Quadratic Formula tells you whether the graph of a quadratic function crosses the *x*-axis and, if it does, where the *x*-intercepts are. **13.** Kelsey can use the Quadratic Formula with $a = 1$, $b = 5$, and $c = -5$ to find values for x^2 that solve the equation. The square roots of these values will solve the original equation. **15.** Remind Sage to set the equation equal to 0 before identifying *a, b,* and *c.* Setting the equation equal to 0 shows that $c = 2$, not -2. When using the correct values for *a, b,* and *c,* the solutions will check.
17. $x = -1 \pm i$ **19.** $x = -3$
21. $x = \frac{3}{2} \pm \frac{\sqrt{23}}{2}i$ **23.** 2 real solutions
25. 2 non-real solutions
27. No; when written in standard form, $-5t^2 + t - 1 = 0$, the discriminant is negative, so there are no real solutions. The ball will not reach a height of 5 meters. **29.** $x = 8$ or $x = -12$
31. $x = -\frac{1}{6} \pm \frac{\sqrt{17}}{6}i$ **33.** $k < 12$
35. a. $C = 44.5x^2 - 3.9x + 9013.6$ where *x* represents the number of years after 2012–13. **b.** 2017–2018 **37. a.** Yes **b.** No **c.** No **d.** Yes **e.** No **39. Part A** $V(w) = 7w^2 - 112w + 196$, where *w* represents the width of the original piece of cardboard **Part B** 17.9 in. by 29.9 in.

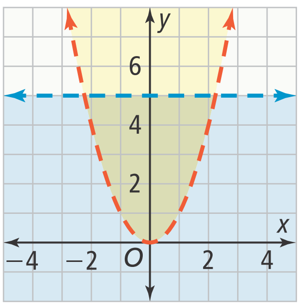
Selected Answers

Topic 2

Lesson 2-7

1. You can solve a linear-quadratic system of equations by graphing the two equations and finding their points of intersection, or by using substitution to create one quadratic equation and solving it. **3.** no solutions **5.** (5.83, 132.76) (−1.83, −12.76) **7.** Both methods are correct, since both reduce the system to one quadratic equation which can then be solved for *x*. **9.** The system must have no solutions, since all *y*-values will be positive for the first equation but $y = -1$ for all *x*'s in the second equation. **11.** 1 solution **13.** Answers may vary. Sample: $m = 1$, $b = 2$ **15.** Answers may vary. Sample: $m = 0$, $b = 0$ **17.** (3, 15)

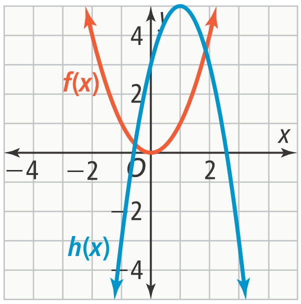
19.



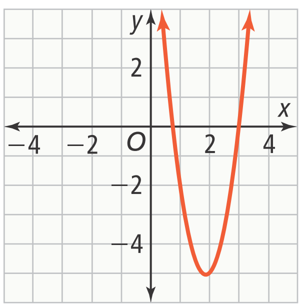
21. $x \approx -0.61$, $x \approx 2.45$ **23.** $x \approx -1.53$, $x \approx 2.28$ **25.** (−4, −3) and (3, 4); I substituted $x + 1$ for *y* in the second equation, solved for *x*, then found the corresponding *y* values. This method was straightforward because the quadratic equation was factorable. **27. a.** no **b.** exactly one **c.** exactly one **d.** exactly one **e.** no **29. Part A** bounce **Part B** 17.5 yards **Part C** Yes; if you measure 180 yards along the hill, that is a smaller distance horizontally than 180.

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** vertex form **5.** complex number **7.** completing the square **9.** The graph is to be translated 1 unit right and 5 units up. It will open downward and be vertically stretched.



11. vertex: (4, −3), axis of symmetry: $x = 4$, minimum: $y = -3$, domain: $(-\infty, \infty)$, range: $(-3, \infty)$
13. $f(x) = -1.5(x - 1)^2 + 5$
15. Sample: (3, 11), (4, 5), (6, 5)
17. vertex: (1.875, −5.063), *y*-intercept: (0, 9)

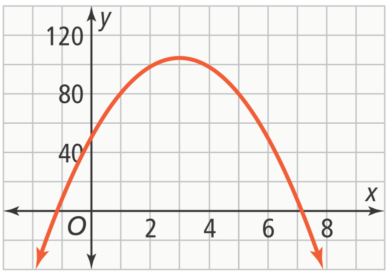


Selected Answers

Topic 2

19. $y = 3x^2 - 19x + 14$

21.

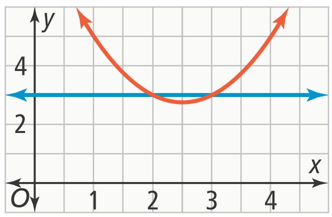


23. $x = 2$ or $x = 5$ **25.** $x = 0.8$ or $x = 3$
27. $x < -7$ or $x > -4$
29. $y = 2(x - 2.5)(x - 8)$ or $y = 2x^2 - 21x + 40$ **31.** $13i$
33. $-\frac{4}{5} - \frac{7}{5}i$ **35.** $5.625 - 3.125i$ amps
37. $(x - 3.5)^2 = 22.75$ **39.** $x = -7 \pm \sqrt{43}$
41. $x = 7.5 \pm \sqrt{44.25}$ **43.** about \$51.39 or \$408.61 **45.** $x = \frac{-5 \pm \sqrt{17}}{2}$

47. $x = \frac{5 \pm \sqrt{253}}{6}$ **49.** 2 nonreal solutions

51. According to the discriminant, the equation has no real roots, so it does not cross the *x*-axis. The graphs of all quadratic functions eventually cross the *y*-axis.

53. 2



55. (–3, 0), (1, 8) **57.** about 1.62 meters above the archer, 3.49 seconds later

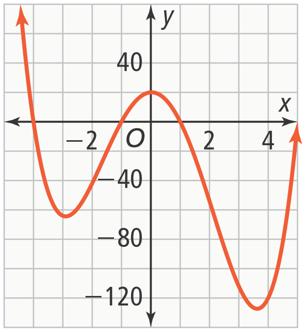
Selected Answers

Topic 3

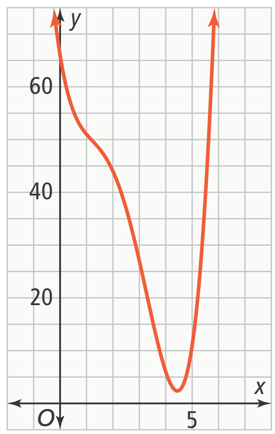
Lesson 3-1

1. The degree and leading coefficient of a polynomial function help determine its end behavior. The end behavior, turning points, and *x*-intercepts help you sketch a graph. **3.** Write the polynomial function in standard form. The leading coefficient is the coefficient of the first term when written in standard from. **5.** 5 **7.** 1 **9.** between –4 and –2, between –2 and 1, between 1 and 3 **11.** approximately (–3, –25), (2, –25) **13.** Check students' graphs. **15.** No; the end behavior of the function when *x* approaches infinity is negative infinity. The revenue earned by the company will be negative. **17.** average rates of change: 1, 17; The average rate of change is much greater over the interval [2, 4]. **19.** $f(x) = 10x^7 - x^4 - 7x^3 + 8x^2$; 7; 4; 10 **21.** The leading coefficient, –1, is negative, so the graph falls to the right. The degree is 5, which is odd, so the end behaviors are different. As *x* becomes infinitely positive, the *y*-values approach $-\infty$. As *x* becomes infinitely negative, the *y*-values approach $+\infty$. **23.** The leading coefficient is negative, so the graph opens downward. The degree is 6, which is even, so the end behaviors are the same. As *x* approaches positive or negative infinity, the *y*-values approach $-\infty$.

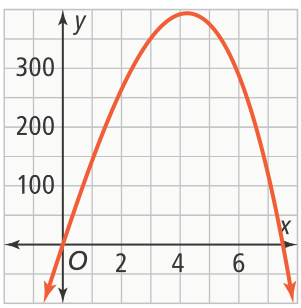
25. zeros: $x = -4$, $x = -1$, $x = 1$, $x = 5$; turning points between –4 and –1, between –1 and 1, between 1 and 5



27. According to the graph, the depth of the lake should reach a relative minimum between $x = 4$ and $x = 5$, within the year 2020.



29. The *x*-intercepts 0 and 7.5 are values of *x* for which the volume is 0. The function has a maximum when *x* is approximately 4. So the container has a maximum volume when height ≈ 4 in., length ≈ 14 in., and width ≈ 7 in.



31. E

Selected Answers

Topic 3

Lesson 3-2

1. To add polynomials, group like terms and combine like terms. To subtract polynomials, distribute the factor of −1 and combine like terms. To multiply polynomials, apply the Distributive Property and then combine like terms. **3.** Polynomials are written in standard form to make it easier to see if two polynomials are equal.
5. $-2a^3 - a^2 - 5a + 3$
7. $14a^3 - 31a^2 + 11a + 6$
9. $V = 3w^3 + 12w^2$
11. As $x \to \infty$, $y \to -\infty$, and as $x \to -\infty$, $y \to \infty$; Answers may vary. Sample: For example, if $f(x) = 3x^2 + 2$ and $g(x) = -4x^3 + 2x^2 - 8$, then $P(x) = -4x^3 + 5x^2 - 6$. **13.** 2, 1, or 0; If the second-degree terms have the same coefficient, the degree of the difference depends on the remaining first- or zero-degree terms. **15.** Sample: no; $(6y^4 - 8y^3 + 24y) \div 4y^2 = \frac{3}{2}y^2 - 2y + 6y^{-1}$. The quotient is not a polynomial by definition, because the exponent of the variable in the last term is not a whole number. **17.** If the leading coefficient is positive, then as $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to -\infty$. If the leading coefficient is negative, then as $x \to \infty$, $y \to -\infty$, and as $x \to -\infty$, $y \to \infty$. **19.** $6y^4 + 3y^3 - 7y^2 + 7y + 15$
21. $-20x^3y + 36x^2y^2 + 4xy^3$
23. $-z^3 + 5z^2 + 41z - 45$
25. $R(x) = (13{,}500 - 500x)(8 + x) = -500x^2 + 9{,}500x + 108{,}000$
27. c. The y-coordinate of the relative maximum represents the maximum volume that can result from cutting four squares from the corners of the sheet metal, and the x-coordinate of the relative maximum represents the measure of the side of the squares that could be cut from the sheet metal in order to produce the maximum volume.
29. $S(x) = 0.028x + 18$ **31.** d

Lesson 3-3

1. First determine if an expression contains a difference of squares, square of a sum, difference of cubes, or sum of cubes. Then identify the squares or cubes. Finally, substitute the squares or cubes into the identity to simplify a calculation. **3.** The coefficients of the terms of the binomial expansion follow the pattern in Pascal's Triangle. **5.** 10, because the first term of expansion is C_0a^5, the expansion will use the fifth row of Pascal's Triangle. Since C_3 represents the coefficient of the fourth term in the expansion, it is the fourth number in the fifth row of Pascal's Triangle, 10. **7.** $4x^2 - 64y^2$
9. $(6a^3 + 2b)(6a^3 - 2b)$
11. $(m^3 + 3n^2)(m^6 - 3m^3n^2 + 9n^4)$
13. $135a^4$ **15.** $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$
17. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$ **19.** Emma took the square root of the exponents. She should have divided the exponents by 2. Factored correctly, $625g^{16} - 25h^4 = (25g^8 + 5h^2)(25g^8 - 5h^2)$. **21.** 1

Selected Answers

Topic 3

23. $n = 6$ **25.** $n + 1$ terms; based on the pattern in Pascal's Triangle, each row has 1 more term than the row number. **27.** $x^4 - y^4 =$
$(x - y)(x + y)(x^2 + y^2)$ Multiply the first two factors using the Difference of Squares.$= (x^2 - y^2)(x^2 + y^2)$
Multiply remaining factors using the Difference of Squares.
$= x^4 - y^4$
29. $x^2 + 12x + 36$ **31.** $4x^2 - 25$
33. $x^4 + 2x^2y^6 + y^{12}$ **35.** $36 - 12y^3 + y^6$
37. 9,991 **39.** 225
41. $(x^3 - 2)(x^6 + 2x^3 + 4)$ **43.** $(x^2 - 3y)(x^4 + 3x^2y + 9y^2)$
45. $(6 + 3y^4)(36 - 18y^4 + 9y^8)$
47. $\left(\frac{1}{4}x^3 + 5y^2\right)\left(\frac{1}{4}x^3 - 5y^2\right)$ **49.** 1,125
51. 504 **53.** $32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5$ **55.** $x^8 + 4x^6 + 6x^4 + 4x^2 + 1$ **57.** $x^{18} + 6x^{15}y^2 + 15x^{12}y^4 + 20x^9y^6 + 15x^6y^8 + 6x^3y^{10} + y^{12}$ **59.** $64m^6 + 384m^5n + 960m^4n^2 + 1{,}280m^3n^3 + 960m^2n^4 + 384mn^5 + 64n^6$ **61.** $27x^3 - 5.4x^2 + 0.36x - 0.008$
63. $m^6 + \frac{3}{2}m^4n + \frac{3}{4}m^2n^2 + \frac{1}{8}n^3$
65. $x^3 - 27$

67. yes; yes; no

69. Part A 25% **Part B** To find the probability that the basketball player will make exactly 7 out of 10 free throws, find $C_3p^7q^3$, where C_3 is the fourth term of the tenth row of Pascal's Triangle; p is the probability of success, 0.8; and q is the probability of failure, 0.2. The expression is $120(0.8)^7(0.2)^3$.
Part C 20%

Lesson 3-4

1. You can use long division or synthetic division to divide polynomials. The process is similar to long division with numbers: ask how many times the divisor goes into the quotient, write the partial quotient, multiply, and subtract. Then bring down the next term(s) and repeat the process until what is left after subtraction has a smaller degree than the divisor. That is then the remainder.
3. The dividend, $P(x)$, is equal to the product of the quotient, $Q(x)$, and the divisor, $D(x)$, plus the remainder, $R(x)$. So, $P(x) = Q(x) \bullet D(x) + R(x)$.
5. $x^2 - 5x - 6 - \frac{23}{x - 3}$
7. The remainder is 0, so $x + 9$ is a factor of $P(x) = x^3 + 11x^2 + 15x - 27$; $P(x) = (x + 9)(x^2 + 2x - 3)$ **9.** First use synthetic division to divide $x^4 + 2x^3 - 16x^2 - 2x + 15$ by $x - 3$. The remainder is 0, so $x - 3$ is a factor of $x^4 + 2x^3 - 16x^2 - 2x + 15$. The quotient is $x^3 + 5x^2 - x - 5$. Now use synthetic division to divide $x^3 + 5x^2 - x - 5$ by $x + 5$.

−5	1	5	−1	−5
		−5	0	5
	1	0	−1	0

The remainder is 0, so $x + 5$ is a factor of $x^3 + 5x^2 - x - 5$, and therefore is a factor of $x^4 + 2x^3 - 16x^2 - 2x + 15$.
11. If the degree of $R(x)$ is greater than or equal to the degree of $d(x)$, then you are not finished dividing. If the degree of $R(x)$ is less than the degree of $d(x)$, then the division is complete. **13.** $n = 5$

Selected Answers

Topic 3

15. $x^2 + 6x + 5$ **17.** $3x + 7 + \frac{28x + 9}{x^2 - 3x}$
19. $x^3 + 4x^2 - 9x - 36$ **21.** $x^4 - 3x^2 + 6x - 11 + \frac{21}{x + 2}$
23. $f(9) = (9)^4 - 6(9)^3 - 33(9)^2 + 46(9) + 75 = 3$

$$\begin{array}{r|rrrrr} 9 & 1 & -6 & -33 & 46 & 75 \\ & & 9 & 27 & -54 & -72 \\ \hline & 1 & 3 & -6 & -8 & 3 \end{array}$$

The remainder is 3, and since $f(9) = 3$, the Remainder Theorem is verified.
25. 52 **27.** 135 **29.** The remainder is not 0, so $x + 4$ is not a factor of $P(x) = 4x^4 - 9x^3 - 7x^2 - 2x + 25$.
31. The remainder is 0, so $2x + 3$ is a factor of $P(x) = 2x^3 + 3x^2 - 8x - 12$; $P(x) = (2x + 3)(x^2 - 4)$ **33.** Use synthetic division to divide $x^3 + 14x^2 + 57x + 72$ by $x + 3$. The quotient is $x^2 + 11x + 24$. Factor $x^2 + 11x + 24$ to get $(x + 3)(x + 8)$. So, $x + 3$ and $x + 8$ are possible dimensions for the length and height of the DVD stand. **35.** B, D
37. Part A $x^3 + x^2 + x + 1$; $x^4 + x^3 + x^2 + x + 1$; $x^5 + x^4 + x^3 + x^2 + x + 1$ **Part B** When $x^n - 1$ is divided by $x - 1$, the quotient is $x^n - 1 + x^n - 2 + x^{n-3} + \ldots + x + 1$. **Part C** $x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

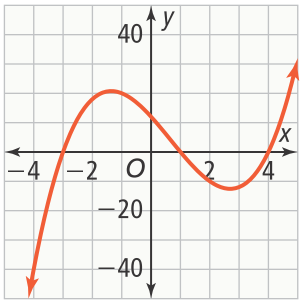
Lesson 3-5

1. The zeros of polynomial functions show where the polynomial either crosses or touches the x-axis. The sign of the values of the function between the intervals created by the zeros determines whether the function is above or below the x-axis. Additionally, the signs of the values of the function to the left and right of the least and greatest zeros determine the end behavior of the graph. **3.** Graph the function using a graphing calculator or graphing program. Use the graph to estimate one of the zeros of the function. Use synthetic division to confirm that value is a zero and to factor the polynomial. Use the Quadratic Formula to find the remaining zeros. **5.** $P(x) > 0$ for $3 < x < 7$, so the company should make between 30,000 and 70,000 LED light bulbs. **7.** Tonya neglected to use synthetic division to factor the polynomial in order to find its complex roots, $-1 + 2i$ and $-1 - 2i$. **9.** When a graph has a multiplicity of a zero that is even, the graph only touches the x-axis, and turns back without crossing. That never occurs in the graph of this polynomial function. **11.** $(-4, -12)$, $(2, -12)$, $(4, -12)$

Selected Answers

Topic 3

13.



15. −5, 1, 5; The graph crosses the x-axis at −5, 1, and 5. **17.** The zeros of the polynomial function are 3, $-1+i$, and $-1-i$. **19.** $x=-5$, $x=2$ **21.** $x=0$, $x=3$, $x=2i$, $x=-2i$ **23.** $x<-2$ or $-\frac{1}{2}<x<\frac{1}{2}$ **25. a.** The domain is $0<t\leq 10$. **b.** The zeros are 0 and 10. They represent the times when the projectile is on the ground. **c.** The vertex is (5, 122.5), which means at $t=5$ seconds, the height of the projectile was 122.5 meters.
27. a. $x(x^2+2x-8)=x(x+4)(x-2)$ **b.** The zeros are $x=0$, $x=-4$, and $x=2$. **c.** $(x-2)$ represents the height, and $(x+4)$ represents the length. **d.** width = 6 in., height = 4 in., and length = 10 in. **29.** C

Lesson 3-6

1. The possible rational roots, or zeros, of a polynomial equation with integer coefficients must come from the list of all numbers of the form $\pm\frac{\textit{factor of constant term}}{\textit{factor of leading coefficient}}$. The degree of a polynomial gives the number of roots, but that might include irrational or complex roots in addition to rational roots.

3. By the Conjugate Root Theorems, $-2i$ and $-\sqrt{7}$ are also roots. A quintic function has 5 roots. Because 4 roots have been accounted for, there is 1 more root. **5.** possible roots: $\pm\frac{1}{1}$, $\pm\frac{2}{1}$, $\pm\frac{3}{1}$, $\pm\frac{4}{1}$, $\pm\frac{6}{1}$, $\pm\frac{9}{1}$, $\pm\frac{12}{1}$, $\pm\frac{18}{1}$, $\pm\frac{36}{1}$; rational roots: −4, −3, 3 **7.** possible roots: $\pm\frac{1}{1}$, $\pm\frac{1}{2}$, $\pm\frac{1}{4}$, $\pm\frac{2}{1}$, $\pm\frac{2}{2}$, $\pm\frac{2}{4}$; rational roots: -2, $-\frac{1}{2}$, $\frac{1}{2}$ **9.** $1-\sqrt{11}$ and $-3-\sqrt{17}$
11. $12-5i$ and $6+\sqrt{13}$ **13.** sometimes; The factors of a_0, or 6, are 1, 2, 3, and 6. The factors of a_n, or 5, are 1 and 5. So, one possible factor is $\frac{3}{1}$, or 3.
15. Irrational roots come in pairs. Either the student incorrectly calculated $\sqrt{3}$ to be a root, or one of the real roots should be $-\sqrt{3}$. **17.** The equation $2x^2-10=0$ can be written as $2(x^2-5)=0$, which has two irrational roots, $\sqrt{5}$ and $-\sqrt{5}$, not $\sqrt{10}$ and $-\sqrt{10}$.
19. When $b^2-4ac>0$, there are two distinct real roots. When $b^2-4ac=0$, there is one real root with multiplicity 2.
When $b^2-4ac<0$, there are two distinct complex roots. **21.** $\pm\frac{1}{1}$, $\pm\frac{1}{2}$, $\pm\frac{3}{1}$, $\pm\frac{3}{2}$, $\pm\frac{5}{1}$, $\pm\frac{5}{2}$, $\pm\frac{9}{1}$, $\pm\frac{9}{2}$, $\pm\frac{15}{1}$, $\pm\frac{15}{2}$, $\pm\frac{45}{1}$, $\pm\frac{45}{2}$
23. $\frac{1}{1}$, $\pm\frac{1}{2}$, $\pm\frac{1}{4}$, $\pm\frac{1}{8}$, $\pm\frac{2}{1}$, $\pm\frac{2}{2}$, $\pm\frac{2}{4}$, $\pm\frac{2}{8}$, $\pm\frac{3}{1}$, $\pm\frac{3}{2}$, $\pm\frac{3}{4}$, $\pm\frac{3}{8}$, $\pm\frac{6}{1}$, $\pm\frac{6}{2}$, $\pm\frac{6}{4}$, $\pm\frac{6}{8}$
25. $\{4, -2+3i, -2-3i\}$
27. $\{6i, -6i, \sqrt{2}, -\sqrt{2}\}$ **29.** yes
31. $P(x)=x^4-6x^3+79x^2-486x-162$
33. a. $0=2x^3+7x^2+7x-268$
b. $\pm\frac{1}{1}$, $\pm\frac{1}{2}$, $\pm\frac{2}{1}$, $\pm\frac{2}{2}$, $\pm\frac{4}{1}$, $\pm\frac{4}{2}$, $\pm\frac{67}{1}$, $\pm\frac{67}{2}$, $\pm\frac{134}{1}$, $\pm\frac{134}{2}$, $\pm\frac{268}{1}$, $\pm\frac{268}{2}$

Selected Answers

Topic 3

c. 4; By using synthetic division, another factor can be found after dividing the polynomial by $x - 4$. There are no real zeros for the factor, which is a quadratic factor, because the discriminant is negative.
d. length: 5 in.; width:6 in.; height: 9 in. **35.** 9 video game consoles **37.** A

Lesson 3-7

1. If the function is even, then its graph will be symmetric about the y-axis, and if the function is odd, its graph will be symmetric about the origin. The transformations can be used to determine a function's dilation, reflection, and horizontal and vertical translations. **3.** The function is shifted to the right 1 unit.
5. odd **7.** Substitute points from the graph into the equation to verify that they satisfy the equation.
9. To determine a translation, look at the point where the graph is least steep. That point is at the origin in the parent graph, so the coordinates of that point in the new graph show the translation. To determine a reflection, look at the end behaviors. If the end behavior of the new function matches that of the parent function, there is no reflection. If not, look to see whether the reflection is across the x- or y-axis. To determine a stretch or compression, look at the y-values of a pair of points one x-unit apart in the parent and in the new function. If the ratio of the y-values in the parent is different from the ratio of y-values in the new graph, then there is a stretch or compression. The ratio of the two ratios gives its value.

11. Because $g(x) \neq g(-x)$, the function is not even. Because $g(-x) \neq -g(x)$, the function is not odd. **13.** odd **15.** neither **17.** parent function: $y = x^3$; Adding 1 (before calculating the cube) shifts the graph to the left 1 unit; multiplying by 3 stretches the translated graph vertically; subtracting 2 shifts the graph down 2 units. **19.** $f(x) = -(x - 1)^3$
21. $V(x) = 81x^3 + 108x^2$
23. $V(x) = 1{,}728x^3 + 288x^2 + 12x$

25.

I	vertical stretch	**A.** 7
II	shift to the left	**B.** 5
III	shift to the right	**C.** 3
IV	shift upward	**D.** 2
V	shift downward	**E.** 1

27. Part A vertex: maximum altitude (in feet) reached by the ball; y-intercept: the height from which the ball is thrown; x-intercept: the time (in seconds) it took the ball to hit the ground **Part B** It will shift the graph up by 4 units, or 4 ft. **Part C** 0.102 s

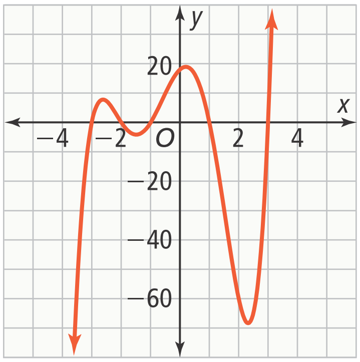
Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** leading coefficient **5.** Pascal's Triangle

Selected Answers

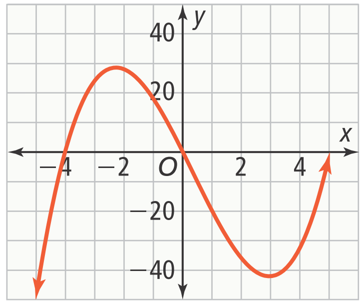
Topic 3

7. Binomial Theorem **9.** zeros: $x = -3$, $x = -2$, $x = -1$, $x = 1$, $x = 3$; turning points between -3 and -2, between -2 and -1, between -1 and 1, between 1 and 3



11. The polynomial function has an odd degree and a negative leading coefficient. **13.** $-3x^3 + 10x^2 - x - 10$
15. $63x^3 + 326x^2 + 53x - 10$
17. $49x^2 - 16$
19. $(3x + y^2)(9x^2 - 3xy^2 + y^4)$
21. $x^5 + 25x^4y + 250x^3y^2 + 1{,}250x^2y^3 + 3{,}125xy^4 + 3{,}125y^5$
23. $4x^3 + 8x^2 + 18x + 8$
25. $x^3 + 8x^2 + 31x + 91$, R290
27. $4x^2 + 2x - 6$ in.

29.



31. The inequality is true for $-3 < x < -2$ and $x > 2$.
33. $Q(x) = x^4 - 2x^3 + 45x^2 - 98x - 196$ **35.** length: 6 ft, width: 11 ft, height: 12 ft **37.** odd
39.

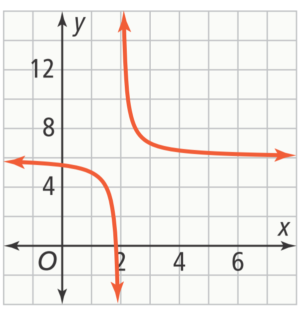
$$\begin{aligned} V(x) &= 10x(10x + 10)(10x - 20) \\ &= (100x^2 + 100x)(10x - 20) \\ &= 1000x^3 - 2000x^2 + 1000x^2 \\ &\quad -2000x \\ &= 1000x^3 - 1000x^2 - 2000x \end{aligned}$$

Selected Answers

Topic 4

Lesson 4-1

1. Sample: Inverse variation can be represented in tables, equations, and graphs. Transformations of the reciprocal function have the form $y = \frac{a}{x-h} + k$. When graphing them, be sure to identify the x- and y-asymptotes. **3.** At the x-axis $x = 0$, but $y = \frac{1}{x}$ is undefined at $x = 0$. **5.** When $x = 4$, $y = \frac{1}{2}$. **7.** $y = \frac{1{,}215}{x}$; 5.4 gallons left after 225 mi **9.** As the x-values increase, the y-values also increase. **11.** The points (2, 4) and (–2, –4) are not correct. The labels of these points should be (2, 2.5) and (–2, –2.5). **13.** $k = xy$ **15.** Yes; as the x-values increase, the y-values decrease. Also the product of each pair of x- and y-values is 60. **17.** 800 kHz/s **19.** vertical asymptote: $x = 2$; horizontal asymptote: $y = 6$; domain: the set of real numbers with $x \neq 2$; range: the set of real numbers with $y \neq 6$

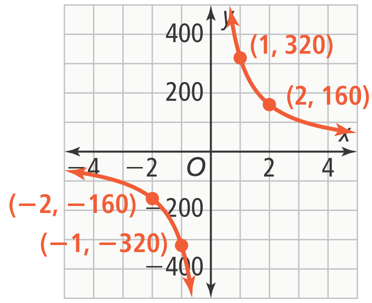


21. a. $n = \frac{320}{5}$

b.

Game size (GB), s	1.0	2.5	3.0	4.0
Number of games, n	320	128	106.7	80

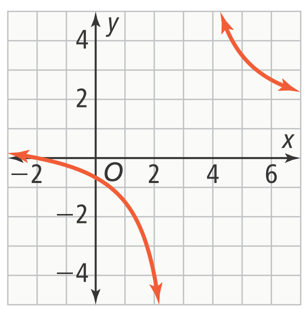
c.



23. B **25. Part A** $d = 66.5t$; 465.5 mi **Part B** $g = \frac{18}{t}$; ≈ 2.57 gal

Lesson 4-2

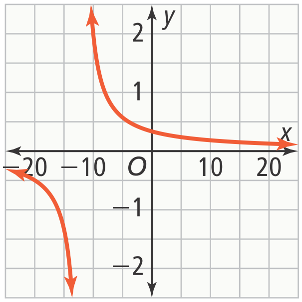
1. Vertical asymptotes may occur when the denominator equals 0. Use long division to rewrite the function. Use the quotient to identify the horizontal asymptotes. Sketch the asymptotes. Then make a table of values to determine behavior near the asymptotes. **3.** Ashton found the ratio of the leading terms. But the degree of the numerator and denominator are not the same, so the horizontal asymptote is not the ratio of the leading terms. The horizontal asymptote is at $y = 0$.
5. vertical asymptote: $x = 3$; horizontal asymptote: $y = 1$



Selected Answers

Topic 4

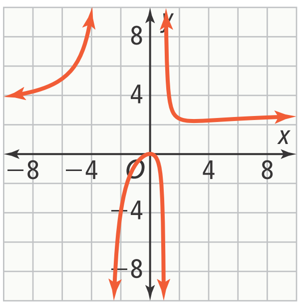
7.



9. Juanita incorrectly used the zeros of the numerator to find the horizontal asymptote. The horizontal asymptote is the ratio of the leading terms. Because the numerator and denominator are polynomials with the same degree, the horizontal asymptote is at $y = 1$.

11. a. Sample: $f(x) = \dfrac{3x^2}{x^2 + 2x - 3}$

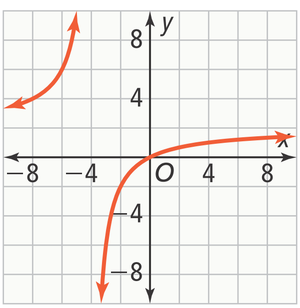
b.



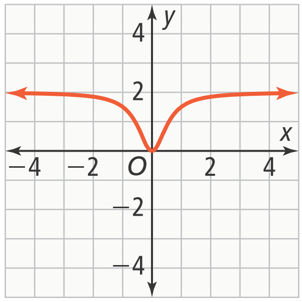
c. Yes, if a graph has other vertical or horizontal asymptotes in additional to those given, the graph would be different. **13.** $f(x) = 2 - \dfrac{8}{x + 4}$;

vertical asymptote: $x = -4$;

horizontal asymptote: $y = 2$

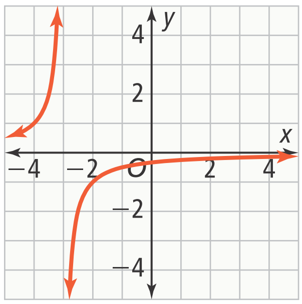


15. $f(x) = 2 - \dfrac{2}{3x^2 + 1}$; vertical asymptote: none; horizontal asymptote: $y = 2$

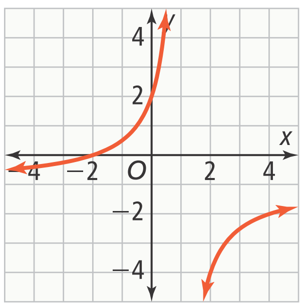


17. vertical asymptote: $x = -\frac{1}{2}$ and $x = \frac{1}{2}$; horizontal asymptote: $y = \frac{3}{4}$
19. vertical asymptote: $x = -2$ and $x = 2$; horizontal asymptote: $y = 0$

21.

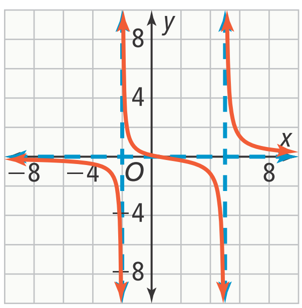


23.



25. day 2 and day 64

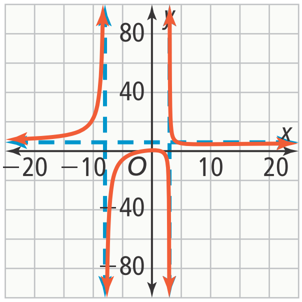
27.



Selected Answers

Topic 4

29.

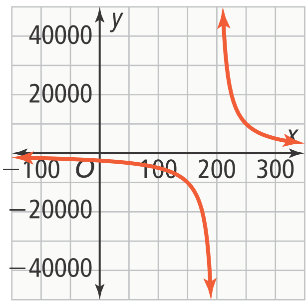


31. a. $f(x) = \dfrac{0.1x + 500{,}000}{x - 200}$

b. vertical asymptote: $x = 200$

c. horizontal asymptote: $y = 0.1$

d.



e. The vertical asymptote means that if 200 or fewer CDs are downloaded, no CDs will be sold, since the first 200 downloads were for testers. The horizontal asymptote means that the average cost will never be less than \$0.10 since this is the manufacturing cost of each CD. **33.** A, D **35. Part A** $y = 0$, $y = 0$, none, none **Part B** When the degree of the numerator is less than the degree of the denominator, there is a horizontal asymptote at $y = 0$. **Part C** When the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

Lesson 4-3

1. The steps and operations used to multiply and divide rational expressions are similar to those used with fractions.

3. The student was supposed to multiply by the reciprocal. The student also did not place any restrictions on the domain.

5. $\frac{x-6}{x-3}$ for all x except -6 and 3

7. $\dfrac{3(x+1)}{2(x+2)}$ for all $x > 0$ **9.** The student cannot cancel the x's in $x + 2$ and $x - 1$ because only common factors, not terms, can be canceled. **11.** Graph them as functions and see that they are the same. When $x = 0$, the function $y = \frac{-6x^2 + 21x}{3x}$ is undefined because the denominator of the first expression equals 0. **13.** never; when you cross multiply, you get $30x + 66 = 30x + 55$; since $66 \neq 55$, there is no real number x for which that equation is true.

15. Yes; even though the factor of x was canceled out, the original fractions cannot be undefined, so the restriction is necessary.

17. $\frac{x(x-2)}{(x-5)}$ for all x except 5 and -6

19. $\frac{(y-8)}{y}$ for all y except 0 and -3

21. $\frac{x+5}{x-4}$, $x \neq -3, 4$ **23.** $\frac{x+4}{x+1}$ for all x except -3, -2, and -1

25. $2x - 1$ for all x except 0, -5, and 5

27. $\frac{(y+4)(y+2)}{3(y-5)}$ for all y except 4 and 5

29. $\frac{5x+2}{x-3}$ for all x except -3, $\frac{2}{5}$, and 3

31. The height of the prism is $3x + 1$ units. **33.** The length of the base of the parallelogram is $\frac{3}{5}$ units. **35.** B,C

37. Part A 9% **Part B** 4.6%

Selected Answers

Topic 4

Lesson 4-4

1. Find a common denominator, add or subtract the numerators, and simplify, eliminating common factor(s) shared by both the numerator and denominator of the sum or difference. **3.** The student incorrectly used the polynomial expression $x + 7$ as the common denominator, when $x(x + 7)$ should have been used. The correct method is $\frac{5x}{x+7} + \frac{7}{x} = \frac{5x(x)}{x(x+7)} + \frac{7(x+7)}{x(x+7)} = \frac{5x^2 + 7x + 49}{x(x+7)}$. **5.** LCD stands for *least common denominator.* The sum or difference will not be in simplest form if you multiply the numerator and denominator of each expression by more factors than necessary.
7. $(x + y)(x - y)^2$ **9.** $\frac{15x^2 - 2y^3}{20xy^2}$
11. $\frac{9x^2 + 5}{3x}$ units **13.** When rewriting the second expression with a common denominator, the student should have had $2x(x + 2)$ in the numerator rather than $x(x + 2)$. The correct answer is $\frac{2x^2 + 4x + 3}{3(x+2)(x+1)}$. **15.** When $x = -y$, the denominator is zero, which is undefined. **17.** $\frac{4x + 9}{x + 7}$
19. $(x - 6)(x + 1)(x - 1)$ **21.** $\frac{8}{x - 8}$
23. $\frac{-4}{(x+1)(x-1)}$ **25.** $\frac{30(r-1)}{r(r-2)}$; 2.75 h
27. $\frac{3x + 7y}{x - 2y}$ **29.** $\frac{(z-4)(z-7)}{(z-5)(z+6)}$
31. $(x - 5)(x + 4)$ or $x^2 - x - 20$
33. A, D **35. Part A** $f = 4$ cm
Part B $x \approx 3.2$ inches

Lesson 4-5

1. Begin by finding the LCD of all the terms, then multiply each term by the LCD, then simplify. Lastly, substitute the solution into the original equation to see if any of the terms are undefined as a result. If so, that solution cannot be an element of the domain and is considered extraneous. **3.** The student did not ensure that the solution is valid. This solution is extraneous, and there is no solution to this equation.
5. $x = -4$ **7.** 12 km/h **9.** The rate must be positive. **11.** Verify that students' answers are correct. **13.** Sample: $\frac{1}{x-2} + \frac{2}{x+6} = 10$ **15.** $x = \frac{31}{10}$
17. $x = \frac{16}{3}$ **19.** $2\frac{2}{5}$ hours
21. no solution **23.** $x = 10$
25. The speed of the current is 1 mi/h.

27. a.

	Distance	Rate	Time
With Wind	1500	$550 + r$	$\frac{1500}{550 + r}$
Against Wind	1000	$550 - r$	$\frac{1000}{550 - r}$

b. $\frac{1500}{550 + r} = \frac{1000}{550 - r}$; $r = 110$ mi/h
29. 40 and 50 mi/h **31.** B, E
33. Part A 150 gallons **Part B** Using the graphing function of your calculator, enter $2.5 + 0.05x$ for y_1, and $1 + 0.06x$ for y_2. Then use the TABLE function to find the y-values of the function that are equivalent when $x = 150$.

Selected Answers

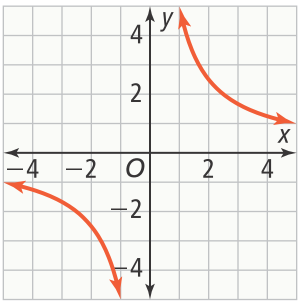
Topic 4

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** rational function **5.** rational expression

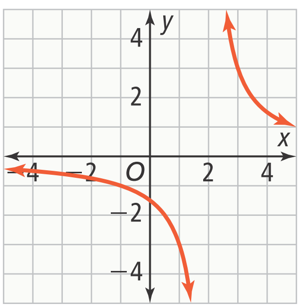
7. compound fraction **9.** $-\frac{1}{2}$

11. domain: all real numbers, $x \neq 0$; range: all real numbers, $y \neq 0$; A horizontal asymptote of the graph is $y = 0$. A vertical asymptote of the graph is $x = 0$.



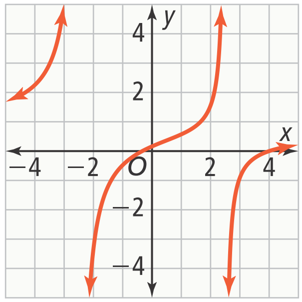
13. 10 cm^3 **15.** vertical asymptotes: $x = -6$, $x = 1$; horizontal asymptote: $y = 0$ **17.** vertical asymptotes: $x = -2$, $x = 8$; horizontal asymptote: $y = 16$

19.



vertical asymptote: $x = 2$;
horizontal asymptote: $y = 0$

21.



vertical asymptote: $x = \frac{5}{2}$, $x = -\frac{5}{2}$;
horizontal asymptote: $y = \frac{3}{4}$

23. 1; $x \neq -4, -2, 3$

25. $\frac{x + 4}{x - 4}$; $x \neq -6, -4, -3, 4, 6$

27. $5x$ units **29.** $\frac{-2(2x + 5)}{(x + 2)(x - 2)}$ **31.** $\frac{-y + 3x}{4y - 5x}$

33. $\frac{12(c + 1)}{(c + 4)(c - 2)}$ **35.** $x = 4$ **37.** $x = -\frac{5}{3}$

39. $x = 15$ **41.** Stacy: 15 hours; Diego: 7.5 h

Selected Answers

Topic 5

Lesson 5-1

1. A radical can be rewritten as a fractional exponent, where the index is the denominator of the exponent. Rational exponents can be used to solve equations with exponents.
3. The index is 5 and the radicand is 125. **5.** No; Anastasia did not apply the Power of a Power Property correctly; $(x^8)^{\frac{1}{4}} = x^{8 \cdot \frac{1}{4}} = x^2$ **7.** $\sqrt[5]{a}$ **9** $b^{\frac{1}{3}}$ **11.** one
13. $x \approx 4.33$ **15.** $3x^4y^2$ **17.** 27
19. Use the volume formula to solve for r. Divide $4{,}186\frac{2}{3}$ by $\frac{4}{3}$ and the result by $\pi \approx 3.14$. This leaves $1{,}000 \approx r^3$. The cube root of 1,000 is 10, so the radius is approximately 10 in. **21.** Yes, they are equal; $\sqrt[3]{x^2}$ and $(\sqrt[3]{x})^2$ can both be written as $x^{\frac{2}{3}}$, so they must be equal. **23.** The negative root can be ignored because the interest rate must be a positive number. **25.** ± 3 **27** 4
29. $16^{\frac{2}{4}} = 16^{\frac{1}{2}}$ **31** $x^{\frac{2}{7}}$ **33.** ± 5 **35.** $2y^3$
37. $\pm 3a^4b^3$ **39.** 5 **41.** 3 **43.** ≈ 205 in.^{3}
45. 12 in. **47.** $\left(\frac{HW}{3{,}600}\right)^{\frac{1}{2}}$ **49.** D

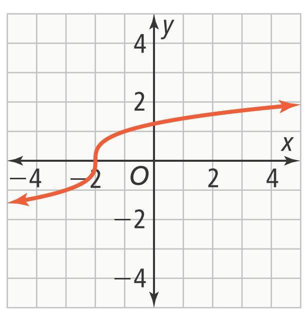
Lesson 5-2

1. Radicals can be rewritten in fractional form, and the laws of exponents can be used to simplify the resulting expressions. **3.** Sample: You can take the cube root of a negative number, but not the square root. **5.** 7
7. $4x^2y^3\sqrt[4]{4x}$ **9.** $-11\sqrt{7}$ **11.** $\frac{\sqrt{6}}{2}$ **13.** 112
15. The exponent $\frac{1}{2}$ applies only to the 5 inside the parentheses; the fractional exponent must be applied before multiplying; correct answer: $20 - 5\sqrt{5}$
17. Sample: $\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ **19.** $12x\sqrt[6]{x}$ **21.** x^3
23. $5z\sqrt[3]{2y^2z}$ **25.** $\frac{4x}{y}$ **27.** $6\sqrt[3]{m}$
29. $3x^6y^5\sqrt{2y}$ **31.** $3x^2\sqrt[3]{x}$
33. $2pq\sqrt[10]{128p^7q^7}$ **35.** $2\sqrt[3]{12}$
37. $10\sqrt[3]{3} - 4\sqrt[3]{9}$ **39.** $41\sqrt{3} - 3\sqrt{6}$
41. $3p + 14\sqrt{5p} - 25$ **43.** -46
45. $-2 - 2\sqrt{3}$ **47.** $\frac{-11 - 8\sqrt{2}}{14}$
49. **a.** $600 + 200\sqrt{3} + 200\sqrt{6}$ m
b. $20{,}000\sqrt{3} + 60{,}000$ m^2
51. $(23 + 10\sqrt{5})yz$ **53.** no; yes; yes; no; yes **55.** **Part A** $r = \sqrt[3]{\frac{3V}{4\pi}} r = \frac{\sqrt[3]{6\pi^2V}}{2\pi}$
Part B about 9.8 in. **Part C** about 33.5 in. tall

Lesson 5-3

1. Vertical stretches and horizontal and vertical translations have the same effect on square root and cube root functions as they have on other types of functions, so you can use what you know about transforming those functions to graph radical functions. **3.** When a is positive, the graph of $f(x) = a\sqrt{x}$ is stretched vertically by a factor of a. When a is negative, the graph of $f(x) = a\sqrt{x}$ is reflected over the x-axis and then stretched vertically by a factor of $|a|$.

5.

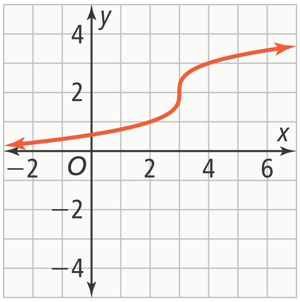


domain: all real numbers;
range: all real numbers

Selected Answers

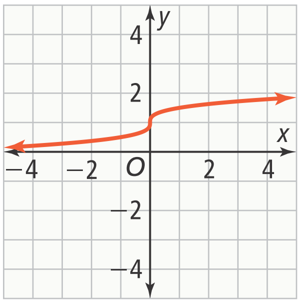
Topic 5

7.



domain: all real numbers; range: all real numbers

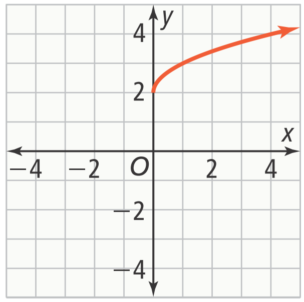
9.



domain: all real numbers; range: all real numbers

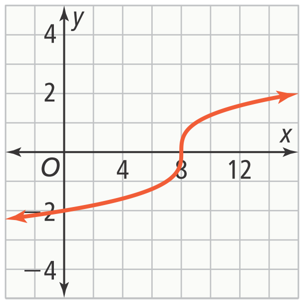
11. The graph of the radical function $h(x) = \sqrt{x + a} + a + b$ is a shift a units left and b units up of the parent function $f(x) = \sqrt{x}$, so the domain of $h(x) = \sqrt{x + a} + a + b$ is $x \geq -a$ and the range is $y \geq b$. The function is increasing over the entire domain. **13.** Helena did not consider the vertical stretch by a factor of 2. The correct radical function is $f(x) = \sqrt[3]{2x - 1}$. **15.** $g(x) = -\sqrt{x}$

17.



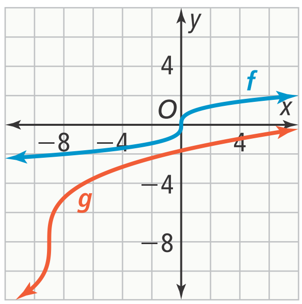
Domain: $x \geq 0$
Range: $y \geq 2$
The function is increasing.

19.



Domain: all real numbers
Range: all real numbers
The function is increasing.

21.



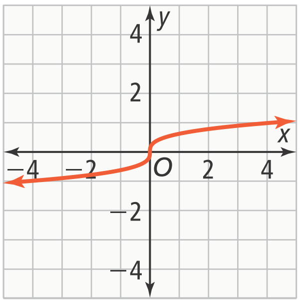
The graph of f is being compressed vertically by a factor of 3 to create the graph of g. The graph of f is shifted 9 units to the left and 8 units down to create the graph of g. The domain and the range are still all real numbers.

Selected Answers

Topic 5

23. $f(x) = 5\sqrt{x+3}$; Vertical stretch by a factor of 5 and translation 3 units left **25.** $f(x) = 2\sqrt{x-6} - 3$; vertical stretch by a factor of 2, translation 6 units right, and translation 3 units down **27.** $f(x) = \sqrt{x-1} + 1$

29. a.



b. The domain of the function is all real numbers. The range of the function is all real numbers. **c.** In this situation, neither the radius nor the volume can be negative, so the domain of this function should be restricted to $r \geq 0$ and the range should be restricted to $V \geq 0$. **d.** about 4.75 in.

31. a. No **b.** Yes **c.** No **d.** Yes **e.** No **33. Part A** When $n = 4$, the domain is $x \geq 0$ and the range is $y \geq 0$. When $n = 5$, the domain is all real numbers and the range is all real numbers. When $n = 6$, the domain is $x \geq 0$ and the range is $y \geq 0$. When $n = 7$, the domain is all real numbers and the range is all real numbers. When $n = 8$, the domain is $x \geq 0$ and the range is $y \geq 0$. **Part B** The domain and range are all real numbers when n is a positive, odd integer. **Part C** The domain is $x \geq 0$ and the range is $y \geq 0$ when n is a positive, even integer.

Lesson 5-4

1. You have to square or cube both sides, use graphs, and be sure to check solutions since solving equations with rational exponents can give extraneous solutions. **3.** Squaring both sides of an equation when solving sometimes produces an extraneous solution. **5.** Raise both sides to the power; cube both sides and then find the square root of the result. **7.** $x = 3{,}125$ **9.** $x = -6$ **11.** $x = 23.\overline{3}$ **13.** $x = 7$ or $x = 3$ **15.** Substitute each potential solution into the original equation. If it makes the equation true, it is a real solution. **17.** The student squared x only instead of 1.98 and x. Here is the correct work:

$$1.98x = \sqrt{58 + y}$$
$$3.92x^2 = 58 + y$$
$$3.92x^2 - 58 = y$$

19. Only if the denominator of the rational exponent is even can there be extraneous solutions. Taking an even root of a negative number gives an extraneous solution, so if the root isn't even, there can't be an extraneous solution. **21.** 125 **23.** 325 **25.** $y = \frac{x^3}{27} - 15$ **27.** $y = 0.0025x^2 + 14.2$ **29.** $x = 3$ **31.** $x = 1$ **33.** $x = 27$ and $x = -32$ **35.** There are no solutions. **37.** $x = -1$ and $x = -\frac{5}{9}$ **39.** 87.27 kg **41.** about 16.61 hours **43.** $\approx 3{,}000$ ft **45.** D

Selected Answers

Topic 5

Lesson 5-5

1. To perform an arithmetic operation on a pair of functions, substitute their rules and perform the operation on them. The domain of the resulting function will be the set of all real numbers for which the original two functions *and* the resulting function are defined. To compose functions, apply one function to the rule of the other. The domain of the resulting function will be the set of all real numbers for which both the composite function *and* the inner function (the first function applied) are defined. **3.** The domain is all real numbers except $x = -3$, for which $g(x)$ is undefined when it is in the denominator. **5.** $3x^2 + 7x$ **7.** $-3x^2 - 3x - 2$ **9.** $\frac{x^2 + 2x + 1}{x - 4}$ for all real numbers x, such that $x \neq 4$ **11.** $18x^2 + 5$ **13.** because there are restrictions on the x-values in the denominator of a fraction such that it cannot be defined **15.** Answers may vary. Sample: When $f(x) = 2x$ and $g(x) = 5x$, $(f \circ g)(x) = 10x$ and $(g \circ f)(x) = 10x$. **17.** When evaluating the nested parentheses in both arithmetic operations and composition of functions, work inside out. **19.** It is possible; Sample: If the graphs of the two linear functions are parallel, then their difference will be a horizontal line; $f(x) = 3x - 4$ and $g(x) = 3x + 2$, so $f(x) - g(x) = -6$, which represents a horizontal line. **21.** $f - g = 2x^2 + 2x - 3$ **23.** $\frac{f}{g} = x - 4$ for all real numbers x, such that $x \neq -7$ **25.** $-28x - 5$ **27.** $-28x + 35$ **29.** $-2x^2 - 2x + 9$ **31. a.** $600h + 500$ **b.** \$5,300 **c.** During an 8-hour period of manufacturing, the cost for the factory to manufacture 240 shovels is \$5,300. **33. a.** subtraction **b.** $p(x) = b(x) - d(x) = -15x + 295$; the domain is the set of all real numbers x such that $0 \leq x \leq 15$. **35.** C

Lesson 5-6

1. In a table, switch the columns x and y. Similarly, in an equation, exchange x and y and solve for y. Graphically, the inverse of a function is its reflection over the line $y = x$. To verify that an inverse function has been found, use composition: $f\left(f^{-1}(x)\right) = x$ and $f^{-1}(f(x)) = x$. **3.** No; the inverse of a function is only a function if each independent value is paired with only one dependent value.
5. $f\left(f^{-1}(x)\right) = -\frac{1}{2}(-2x + 10) + 5 = x - 5 + 5 = x$, $f^{-1}(f(x)) = -2\left(-\frac{1}{2}x + 5\right) + 10 = x - 10 + 10 = x$
7. $f(x)$ and $f^{-1}(x)$ are reflections of each other over the line $y = x$. Therefore, $f(x)$ and $f^{-1}(x)$ are inverses. **9.** The range of $f^{-1}(x)$ is the same as the domain of $f(x) = \sqrt{2x - 3}$ is $2x - 3 \geq 0$ or $x \geq \frac{3}{2}$. So, the range of $f^{-1}(x)$ is $x \geq \frac{3}{2}$.
11. The inverse of a number raised to the 4th power is the 4th root of a number. To find the inverse of $f(x)$, add 1 to both sides. To undo x raised to the 4th power, take the 4th root of both sides. The inverse is $f^{-1}(x) = \sqrt[4]{x + 1}$. The inverse is not a function because each independent value is not paired with only one dependent value.

Selected Answers

Topic 5

13. $a^{-1}(b) = 2\sqrt{b}$;
$a(a^{-1}(b)) = \frac{1}{4}(2\sqrt{b})^2 = \frac{1}{4}(4b) = b$;
$a^{-1}(a(b)) = 2\sqrt{\frac{1}{4}b^2} = 2\left(\frac{1}{2}b\right) = b$

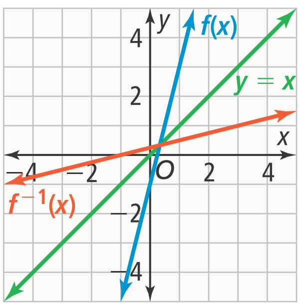
15. x-intercept: $\left(-\frac{1}{2}, 0\right)$, y-intercept: (0, 1); x-intercept of the inverse: (1, 0), y-intercept of the inverse: $\left(0, -\frac{1}{2}\right)$; Sample: the x-intercept of the original function becomes the y-intercept of the inverse by reversing the coordinates, and the y-intercept of the original function becomes the x-intercept of the inverse by reversing the coordinates.

17.

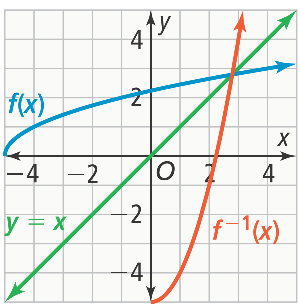
x	–7	6	8	–1	3	7
y	–2	1	0	1	2	3

The inverse is a function.

19. $f^{-1}(x) = \frac{x+1}{4}$; The inverse of f is a function.



21. $f^{-1}(x) = x^2 - 5$; The inverse of f is a function.



23. $f(x) = x^2 - 6x + 9 = (x - 3)^2$, so the graph of the parabola has its vertex at (3, 0). Restrict the domain of f to $x \geq 3$. Then $f^{-1}(x) = \sqrt{x} + 3$, $x \geq 0$.
25. $f(x) = x^2 + 5$, so the graph of the parabola has its vertex at (0, 5). Restrict the domain of f to $x \geq 0$. Then $f^{-1}(x) = \sqrt{x - 5}$, $x \geq 5$.
27. $f^{-1}(x) = x^2 - 6$; domain $x \geq 0$
29. $f^{-1}(x) = x^2 + 9$; domain is $x \geq 0$.
31. $f(g(x)) = \sqrt{\frac{(3x^2 - 4) + 4}{3}} = \sqrt{\frac{3x^2}{3}} = \sqrt{x^2} = x$; $g(f(x)) = 3\left(\sqrt{\frac{x+4}{3}}\right)^2 - 4 = 3\left(\frac{x+4}{3}\right) - 4 = x + 4 - 4 = x$, so f and g are inverses. **33.** $C = \frac{9}{5}F + 32$; 132.8°C
35. a. $r = \sqrt{\frac{V}{\pi h}}$ **b.** $h > 0$ and $V > 0$ because division by 0 is not possible and the height and volume of a coffee can cannot be negative. **c.** 3 in. **37.** B

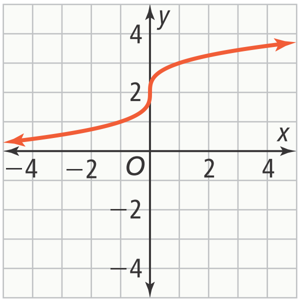
Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** radicand **5.** radical function **7.** extraneous solution **9.** composite function **11.** –625 **13.** $2a^6b^2$ **15.** $z = 4$ **17.** about 5.53 seconds **19.** $\frac{\sqrt[3]{2{,}187m^3}}{3m}$ **21.** $7a^3$ **23.** $81x^2 + 18x\sqrt{2} + 2$ **25.** $12 - \sqrt[3]{4}$ **27.** $-10 - 5\sqrt{5}$ **29.** $-9\sqrt{x}$ **31.** 4 cans

Selected Answers

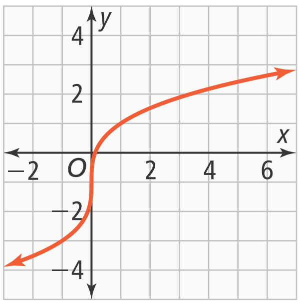
Topic 5

33.



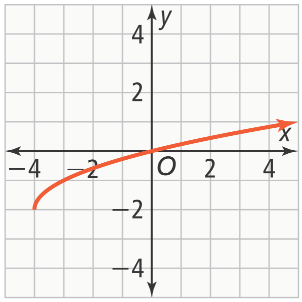
The domain of the function is all real numbers. The range of the function is all real numbers.
The function is increasing over the entire domain.

35.



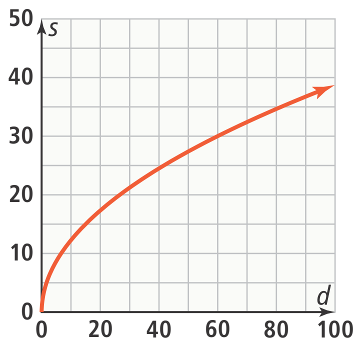
The domain of the function is all real numbers. The range of the function is all real numbers.
The function is increasing over the entire domain.

37.



The domain of the function is $x \geq -4$. The range of the function is $y \geq -2$. The function is increasing over the entire domain.

39.



When $d = 40$, $s = \sqrt{30 \bullet 0.5(40)} \approx 24.49$ mi/h, which is less than 25 mi/h. So, the car was not speeding.

41. $x = 72$ **43.** $x = 81$ **45.** $x = 4$ (extraneous), $x = 14$ **47.** about 10.5 tons **49.** $-6x + 6$ **51.** 11
53. $f(g(78)) = 82.4$; 78 represents the original test score. Two bonus points were added to the original test score. Then a 3% increase was applied to the test score. The new test score is 82.4.
55. $f^{-1}(x) = x^2 + 4$
57. $f^{-1}(x) = (x + 1)^2 - 7$
59. $C = 40h + 50$; $h = \frac{C - 50}{40}$; 2.5 h

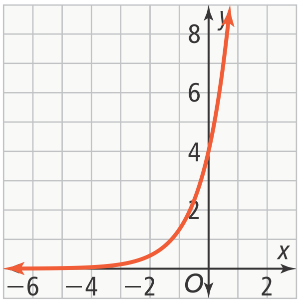
Selected Answers

Topic 6

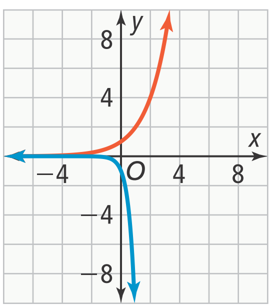
Lesson 6-1

1. The values of equations determine what the key features will be. Graphs illustrate these features. **3.** $\frac{3}{2}$ is greater than 1, so the graph represents exponential growth.

5.



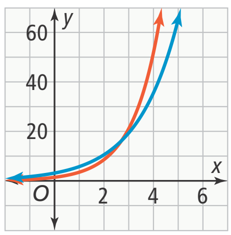
domain: all real numbers; range: $\{y \mid y > 0\}$; y-intercept: 4; asymptote: x-axis; end behavior: As $x \to -\infty$, $y \to 0$. As $x \to \infty$, $y \to \infty$.
7. The graph of $g(x)$ is shifted 3 units to the right. **9.** 4 **11.** The rate is 0.3, so the decay factor is 0.7. **13.** The graph of $g(x)$ is shifted 4 units down and 1 unit to the left. **15.** domain: all real numbers; range: $\{y \mid y > 0\}$; y-intercept 0.75; asymptote: x-axis; end behavior: As $x \to -\infty$, $y \to \infty$. As $x \to \infty$, $y \to 0$.
17. domain: all real numbers; range: $\{y \mid y > 0\}$; y-intercept 7; asymptote: x-axis; end behavior: As $x \to -\infty$, $y \to 0$. As $x \to \infty$, $y \to \infty$. **19.** decay; $1 - \frac{2}{5}$; $\frac{2}{5}$; 40% **21.** growth; 1 + 1; 1; 100%
23. $g(x) = -6(2)$



25. $P(t) = 50(4)^x$; 51,200 bacteria
27. Yes; After 2 years, there will be about 271 zebras left and about 256 lions left. **29.** C

Lesson 6-2

1. Sample: Exponential models can be used to represent how the balances in investment accounts change as they increase. **3.** Sample: Both are exponential functions. For continuously compounded interest, the base is always e, whereas the base of a compound interest function varies.
5. 0.25%; 0.74% **7.** \$2,323.23
9. $y = 26.9(1.27)^x$ **11.** \$8,069.41
13. a. Substitute one of the given points into the model to find the value of a; $y = 7.39e^x$ **b.** Substitute 8 into the model to find $y = 22{,}029$.
15. exponential model: $y = 1.414(2.462)^x$; power model: $y = 3.125(1.837)^x$; exponential model: 314.9; power model: 120.1; The exponential model is a better fit.



17. 7,543.88 **19.** 1,716.79
21. 43,492.51 **23.** $y = 53.0(1.082)^x$
25. Multiply the exponent by $\frac{4}{4}$ so that the model compounds quarterly.
27. Jacinta; Continuously compounded interest increases at a faster rate.
29. \$4,905.04; \$1,405.04 **31.** B, D

Selected Answers

Topic 6

33. Part A \$455.38 **Part B** \$541.78 **Part C** Each month of the second year, she continues to pay interest on more interest.

Lesson 6-3

1. Logarithms are exponents. Logarithms can be evaluated by rewriting them in exponential form. **3.** Common logarithms use the base 10. Natural logarithms use the base *e*.

5. $\log_2\left(\frac{1}{64}\right) = -6$ **7.** $10^{2.301} \approx 200$

9. 3 **11.** 5 **13.** Graph $y = e^x$ and use the graph to find an approximate value of *x* for which *y* is equal to 65. **15.** 3.5; $\log_3 50$ is an expression equivalent to $3^x = 50$, so you need to find the value of *x* for which $y = 50$.

17. yes; $3^{-3} = \frac{1}{27}$ **19.** $\log_4 y = x$

21. $\log_7 y = x$ **23.** $\log_3 6561 = 8$

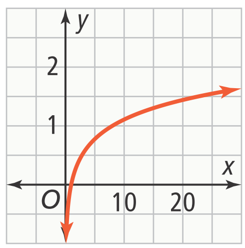
25. $\log_5 1 = 0$ **27.** $10^{-2} = \frac{1}{100}$

29. $e^5 = 148.41$ **31.** –3 **33.** 4 **35.** 3 **37.** undefined **39.** 1.8949 **41.** undefined **43.** 1.0792 **45.** 142 **47.** 2.7964 **49.** 2.8904 **51.** 22 years **53. a.** magnitude 7.9 **b.** 31,009,857,350 kWh **c.** 16,705 kWh **55.** C, D, E **57. Part A** Account 1: $A = 400e^{0.08t}$; Account 2: $A = 500 + 75t$ **Part B** yes; Account 2 is increasing at a constant rate while Account 1 is increasing at an increasing rate, so after a certain point the amount of money in Account 1 will exceed the amount of money in Account 2; in about 20 years.

Lesson 6-4

1. The domain of the logarithmic function is the same as the range of the exponential function, and the range of the logarithmic function is the same as the domain of the exponential function. The logarithmic function has an *x*-intercept of 1, while the exponential function has a *y*-intercept of 1. **3.** The graph of $g(x)$ is the reflection of the graph of $f(x)$ across the *x*-axis. **5.** $g(x) = \ln x + \frac{1}{2}$ **7.** no; The *x*-intercept of the blue function is not equal to the *y*-intercept of the red function. **9.** When rewriting in logarithmic form, the student should have placed the expression $x - 2$ in parenthesis. The correct inverse function is $f^{-1}(x) = \log_5 (x - 2) + 6$. **11.** $g(x) = \log_3 (x - 2) + 1$

13.

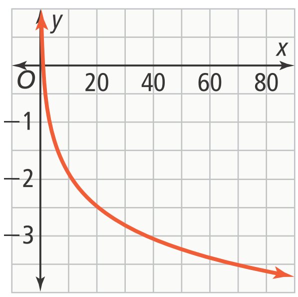


domain: $\{x \mid x > 0\}$; range: all real numbers; *x*-intercept 1; asymptote: *y*-axis; As $x \to 0$, $y \to -\infty$. As $x \to \infty$, $y \to \infty$.

Selected Answers

Topic 6

15.

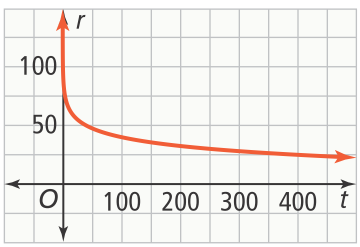


domain: $\{x \mid x > 0\}$; range: all real numbers; x-intercept: 1; asymptote: y-axis; end behavior: As $x \to 0$, $y \to \infty$. As $x \to \infty$, $y \to -\infty$.

17. reflection over the y-axis; asymptote: y-axis (same as parent function); x-intercept: –1 (different from parent function)
19. $f^{-1}(x) = \log_5 x + 3$
21. $f^{-1}(x) = \log_6 x - 7$

23. $f^{-1}(x) = e^{x+1} - 3$ **25.** $t = 2e^{\frac{(y-8{,}000)}{5{,}000}}$; Use this new equation when the altitude of the plane is known but the time since takeoff is not. **27. a.** Check students' work.

x	y
9	65
99	40
200	32.5
300	28
400	24.9



b. $t = 10^{\frac{(r-90)}{(-25)}} - 1$; the number of months since taking the course at which a student has a retention, r

29. C, D, F **31. Part A** magnitude 10

Part B $d = 10^{\frac{(M-2)}{5}}$ **Part C** Astronomers can use this function when the limiting magnitude of a telescope is known to find the diameter of the lens of the telescope.
Part D 199.5 mm

Lesson 6-5

1. The properties of logarithms help to simplify complex logarithmic expressions. **3.** Amanda used the wrong exponent for c. The expression should be $2 \log_4 c + 5 \log_4 d$.
5. $\ln(s^5t^6)$ **7.** Expand the expression and then substitute the approximate values into the expression to find that the approximate value of $\log_3\left(\frac{2}{5}\right)$ is –0.834. **9.** $y = \log\left(\frac{1}{x}\right) = \log 1 - \log x = 0 - \log x = -\log x$ **11.** The student used 2 as an exponent instead of $\frac{1}{2}$. The correct expression is $\log_3 2y^{\frac{1}{2}}$.
13. Let $\log_b(m^n) = x$.

$$b^x = m^n$$
$$(b^x)^{\frac{1}{n}} = (m^n)^{\frac{1}{n}}$$
$$b^{\frac{x}{n}} = m$$
$$\frac{x}{n} = \log_b m$$
$$x = n\log_b m$$

$\log_b(m^n) = n\log_b(m)^n$

15. $\log_6 2 + 5\log_6 m + 3\log_6 n$

17. $\log_2 x - \log_2 5 - \log_2 y$

19. $\log_5\left(6y^{\frac{1}{2}}\right)$ **21.** $\frac{\ln 3}{\ln 8y^3}$

23. $\text{pH} = \log\left(\frac{1}{[H^+]}\right) = \log 1 - \log[H^+] = 0 - \log[H^+] = -\log[H^+]$

Selected Answers

Topic 6

25. 0.898 **27.** 2.807 **29.** 3.135

31. $\left(\frac{\log 11}{\log 5}\right)$; 1.490 **33.** $\left(\frac{\log 30}{\log 2}\right)$; 4.907

35. $\left(\frac{\log 55}{\log 4}\right)$; 2.891 **37. a.** $A = \log(5t^2)$

b. 4,325 people **39.** D

Lesson 6-6

1. Properties of logarithms and exponents help to solve equations in which the terms do not have matching bases. **3.** Use the power property to move the exponent outside the logarithm, divide both sides by the exponent, then rewrite the equation in exponential form and solve.
5. –8.84 **7.** 6 **9.** 13,308 rabbits
11. The terms in the first equation do not have equivalent bases. The terms in the second equation can be manipulated to have the same base. **13.** To solve the equation when $A = 0$, Thomas would need to find the natural log of 0, which is undefined. **15.** –5 is extraneous
17. 0.6833 **19.** –0.4286 **21.** 5
23. 4.5101 **25.** –3.1699 **27.** 7
29. –1; 5 **31.** –1.2749; 6.2749
33. 1.266; –2.766 (extraneous)
35. 4; –1 (extraneous) **37.** 1.303 (extraneous); –2.303 (extraneous)
39. $x \approx 0.003$ and $x \approx 2.153$ **41.** 2045
43. a. 500 **b.** 1,000,000 acres **45.** B

Lesson 6-7

1. If a sequence has a set of numbers with a common ratio, you can create and use an explicit formula in order to extend the sequence or solve a problem.

3. A common difference and common ratio are both the repeated pattern in a sequence of numbers. A common difference is added, while a common ratio is multiplied. **5.** –2; 32, –64, 128 **7.** 3; 64.8, 194.4, 583.2
9. 0.5; 6.25, 3.125, 1.5625 **11.** True; If the first two terms are positive, the common ratio must be positive.
13. Check students' answers. **15.** 59,049

17. yes; $a_n = \begin{cases} 3, \text{ if } n = 1 \\ -5a^{n-1}, \text{ if } n > 1 \end{cases}$

19. yes; $a_n = \begin{cases} 24, \text{ if } n = 1 \\ \frac{1}{3}a_{n-1}, \text{ if } n > 1 \end{cases}$

21. yes; $a_n = \begin{cases} 10, \text{ if } n = 1 \\ 4a_{n-1}, \text{ if } n > 1 \end{cases}$

23. $a_n = 2(-2)^{n-1}$

25. $a_n = \begin{cases} -6, & n = 1 \\ -3a_{n-1}, & n > 1 \end{cases}$

27. 800 bacteria **29.** 6,291,450

31. ≈ –3,355,443 **33.** $\sum_{n=1}^{6} - 7(6)^{n-1}$; –65,317

35. $\sum_{n=1}^{7} 4(-3)^{n-1}$; 2,188 **37.** \$156.94

39. a. 155 people **b.** 1,562 people
41. 7 **43. Part A** \$476.72
Part B \$3,162.00

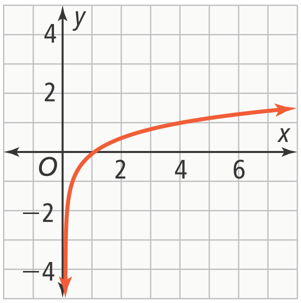
Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** exponential function **5.** Change of Base Formula
7. logarithmic function **9.** domain: all real numbers; range: $y > 0$; y-intercept 2; asymptote: $y = 0$; end behavior: As $x \to -\infty$, $y \to 0$. As $x \to \infty$, $y \to \infty$.

Selected Answers

Topic 6

11. \$13,784.20 **13.** \$13,849.69
15. \$8,801.33 **17.** $f(x) = 21.01(1.37)^x$
19. $\log_4 64 = 3$ **21.** $6^3 = 216$
23. –2 **25.** 2.798 **27.** 9
29. domain: $x > 0$; range: all real numbers; x-intercept: 1; asymptote: $x = 0$; as $x \to 0$, $y \to -\infty$; as $x \to \infty$, $y \to \infty$



31. $f^{-1}(x) = \log_8 x + 2$ **33.** $\log\left(\frac{r^3t}{s^2}\right)$
35. 1.792 **37.** $\frac{\log 200}{\log 5}$; 3.292 **39.** –2
41. –1.151 **43.** 9 **45.** –4; $\frac{9}{2}$ **47.** no
49. $a_n = \frac{1}{8}\left(\frac{3}{2}\right)^{n-1}$ **51.** 1,530
53. –10,230; of the ten terms in each series, the last nine terms in the first series and the first nine terms in the second series are the same, so their difference is 0. This leaves the first term in the first series minus the last term in the second series, which is 10 – 10,240 = –10,230.

Selected Answers

Topic 7

Lesson 7-1

1. Knowing the ratios allows for finding any angle measure or side length in a right triangle when one angle measure and one side length or no angle measures and two side lengths are known. **3.** An identity is a rule that is always true. The tangent of an angle θ is $\frac{\text{opposite}}{\text{hypotenuse}}$. No matter the angle measure, that must always be equal to $\frac{1}{\frac{\text{hypotenuse}}{\text{opposite}}}$, or $\frac{1}{\cot\theta}$.
5. Knowing one ratio makes known the lengths of two out of three sides in the right triangle. The Pythagorean Theorem can be used to find the length of the third side. Once all the side lengths are known, all of the ratios can be found. **7.** $\sin\theta = \frac{3}{7}$ **9.** $\sin\theta = \frac{12}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$ **11.** $\cos\theta = \frac{1}{\sec\theta}$
13. $\csc\theta = \sec(90° - \theta)$ **15.** 20 m
17. Yes, if you have one side and one non-right angle, you can use a trigonometric ratio to find the other sides and angles. If you have two sides, you can use the Pythagorean Theorem to find the third side and then use trigonometric ratios to find the angle measures.
19. The student was given the adjacent side, not the opposite side, so he or she should have used cosine: $\cos 41° = \frac{25}{x}$; 33.1 in. $\approx x$. **21.** If the angles are known, there are infinitely many side lengths that make any of the trigonometric ratios true. For example, sin 30° equals any opposite side and hypotenuse that have a ratio of 0.5. However, if the side lengths are known, each angle can be precisely identified using the appropriate trigonometric ratio.

23. $\sin\theta = \frac{24}{26}$, $\cos\theta = \frac{10}{26}$, $\tan\theta = \frac{24}{10}$, $\csc\theta = \frac{26}{24}$, $\sec\theta = \frac{26}{10}$, $\cot\theta = \frac{10}{24}$
25. $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$ **27.** $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$ **29.** 7 ft
31. $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$
33. $\cos\theta = \sin(90° - \theta)$
35. tan 75° = 3.7, which is approximately 4. So, if someone uses the rule, he or she will have about a 75° angle. A person could have up to a 76° angle by using the rule, which would be just above what is safe. **37.** 195 ft **39.** E

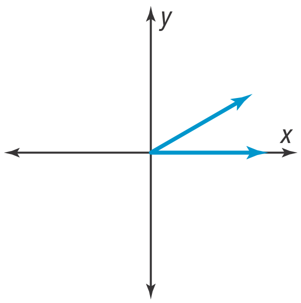
Lesson 7-2

1. by plotting angles on a unit circle and identifying their reference angles
3. A unit circle's center is at the origin, and its radius is 1. **5.** Since radian measures of angles are usually less than or equal to 2π, if the answer is very small or written in terms of π, it is likely in radian mode. **7.** 35° **9.** Quadrant IV
11. –250° **13.** 315° **15.** $\frac{8\pi}{3}$ **17.** Since the radius of a unit circle is always 1, the angle measure in radians is equal to the length of the intercepted arc, divided by 1. Therefore, the angle measure in radians is always equal to the length of the intercepted arc.
19. The sum of the positive angle and the absolute value of the negative angle is 360° or 2π, so |negative angle| + positive angle = 360° or 2π. **21.** 135°, –225°, 495° **23.** 165°
25. 188°

Selected Answers

Topic 7

27.



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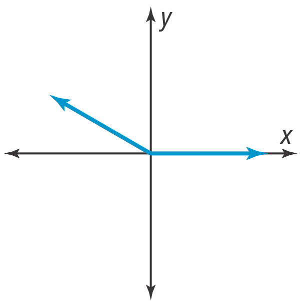

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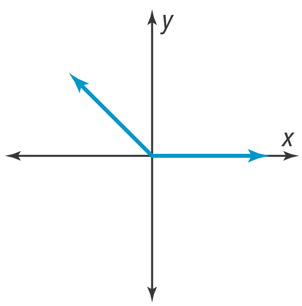

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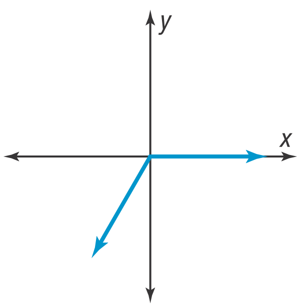
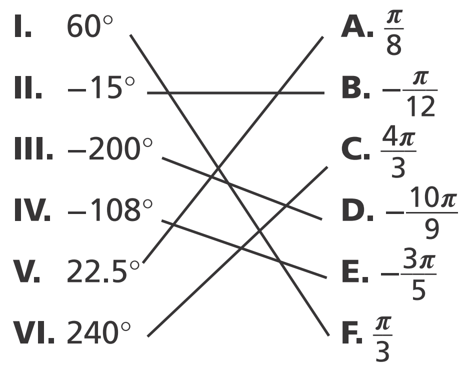


35. 2.58 **37.** 120° **39.** 6,911.5 km (using π); 6,908 km (using 3.14)

41. Substitute the formula for converting radians to degrees into the arc length formula:
length of intercepted arc
$= \left(\frac{\pi}{180^\circ} \times \text{angle measure in degrees}\right) \times \text{radius}.$

43.



45. Part A $\approx$ 94 m **Part B** $\approx$ 0.224 radians

Lesson 7-3

1. The unit circle can be used to evaluate the trigonometric functions. **3.** If an angle is in standard position, the reference triangle for θ includes the acute angle formed by the x-axis and the terminal side of θ. A reference triangle helps you relate all angles to an angle in Quadrant I.

5. $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$; $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

7. $\sin \theta = \frac{3}{5}$ **9.** $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

11. $\sec 135^\circ = -\sqrt{2}$; $\csc 135^\circ = \sqrt{2}$; $\tan 135^\circ = -1$ **13.** Multiply the number of radians by $\frac{180^\circ}{\pi}$. **15.** in Quadrant I, because that is the only quadrant in which the x-coordinate and the y-coordinate of the terminal point are both positive

Selected Answers

Topic 7

17. Infinitely many since any given angle can have an infinite number of coterminal angles. If the angle is measured in degrees, you can keep adding multiples of 360° or −360°; if the angle is measured in radians, you can keep adding multiples of 2π or -2π to it. **19.** $\tan\theta = \frac{y}{x} = -\frac{4}{3}$

21. $\sin 225° = -\frac{\sqrt{2}}{2}$; $\cos 225° = -\frac{\sqrt{2}}{2}$

23. $\sin\frac{29\pi}{4} = -\frac{\sqrt{2}}{2}$; $\cos\frac{29\pi}{4} = -\frac{\sqrt{2}}{2}$

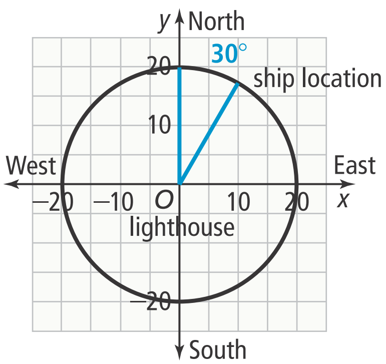
25. $\cos\theta = \frac{7}{25}$ **27.** $\tan 405° = 1$

29. $\sec\frac{13\pi}{4} = -\sqrt{2}$; $\csc\frac{13\pi}{4} = -\sqrt{2}$; $\cot\frac{13\pi}{4} = 1$ **31.** $\sec\frac{-2\pi}{3} = -2$; $\csc -\frac{2\pi}{3} = -\frac{2\sqrt{3}}{3}$; $\cot -\frac{2\pi}{3} = \frac{\sqrt{3}}{3}$

33. about 38 ft **35.** 120°; $\frac{2\pi}{3}$ radians

37. D

39. Part A



Part B 60° **Part C** The coordinates are $(10, 10\sqrt{3})$.

Lesson 7-4

1. For the functions $y = a \sin bx$ and $y = a \cos bx$, the parameter a affects the amplitude and the parameter b affects the period and frequency.

3. The period is the interval of the domain before it repeats. The amplitude is the distance between the midline and the minimum or maximum.

5. period: π; amplitude: 3 **7.** $\frac{1}{2}$

9. The sine function is a periodic function because the outputs repeat after a constant interval of inputs, 2π radians.

11. zero-min-zero-max-zero

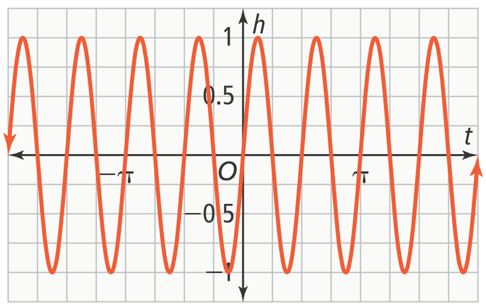
13. domain: $-\infty < x < \infty$; range: $-1 \leq y \leq 1$; period: 2π

15. amplitude: 5; period: 8π

17. $y = 14 \cos \frac{\pi}{3}x$; $\frac{\pi}{3}$

19. a. $\frac{\pi}{2}$ **b.** 1 **c.** $h(t) = \sin(4t)$

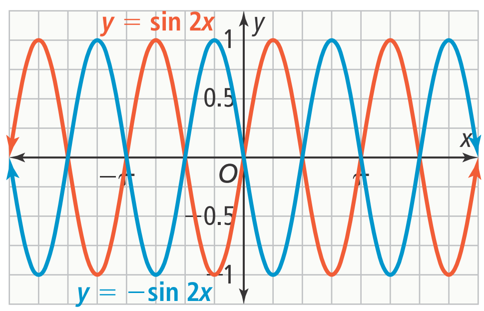
d.



e. 8

21. amplitude: 8; period: 12; frequency: $\frac{1}{12}$; midline: $y = 0$

23. Part A

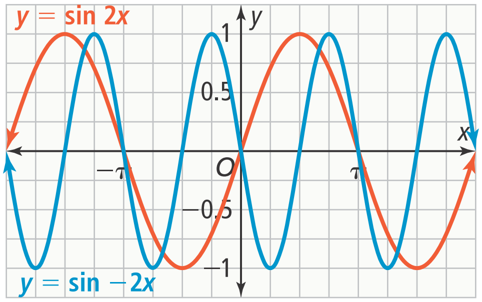


Part B $y = \sin(2x)$ and $y = -\sin(2x)$ are reflections of each other over the x-axis.

Selected Answers

Topic 7

Part C

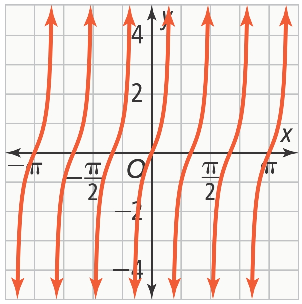


Part D $y = \sin(2x)$ and $y = \sin(-2x)$ are reflections of each other over the *x*-axis. **Part E** When *a* or *b* is replaced with its opposite, the graph is reflected over the *x*-axis.

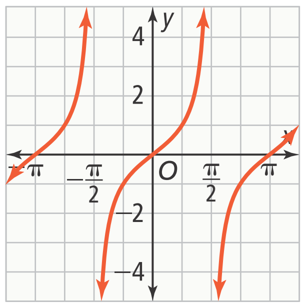
Lesson 7-5

1. Where a function (like sine) is 0, its reciprocal, cosecant, is undefined and has a vertical asymptote. Wherever sine reaches its maximum value at 1, the cosecant will reach its local minimum value of 1; wherever the sine reaches its minimum value of −1, the cosecant will reach its local maximum value of −1. Wherever sine is positive and less than 1, the cosecant will be positive and greater than 1; wherever the sine is negative greater than −1, the cosecant will be negative but less than −1. The same reasoning can be used to describe the cosine function and its reciprocal, secant, as well as the tangent function and its reciprocal, cotangent.
3. The graph of $y = \tan x$ does not have a maximum or minimum value.
5. $\frac{\pi}{3}$ **7.** $y = \tan\left(\frac{1}{2}x\right)$ or $y = \tan\left(-\frac{1}{2}x\right)$

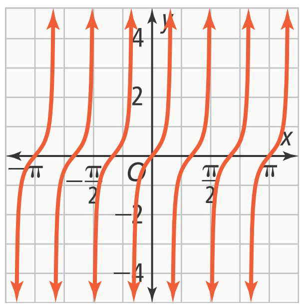
9. That is where its reciprocal function, sine, is 0. **11.** The period for the tangent and cotangent functions is π, whereas the other four trigonometric functions have a period of 2π.
13. $y = \tan 3x$;



15.



17.

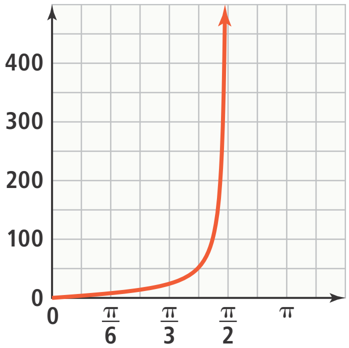


There is a vertical compression that makes the graph look more bent than the parent function $y = \tan x$. The horizontal compression changes the period of the function from π (for $y = \tan x$) to $\frac{\pi}{3}$.

Selected Answers

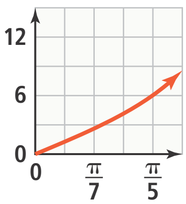
Topic 7

19. a. $h = 20 \tan \theta$
b. domain: $\left(0, \frac{\pi}{2}\right)$; **c.** about 20 ft



21. a. $h = 12 \tan \theta$

b.



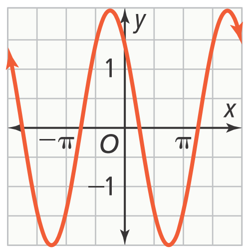
c. ≈ 3.9 ft **23. a.** $\frac{\pi}{2}$ **b.** all real numbers **c.** $-\pi$; 3π **d.** $-\frac{3\pi}{2}$; $\frac{\pi}{2}$ **25. Part A** $L(\theta) = 4 \csc \theta + 6 \sec \theta$ **Part B** 14 ft

Lesson 7-6

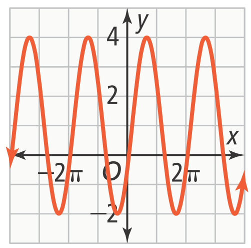
1. You can analyze the graphs of sine and cosine functions to find changes in amplitude, period, phase shift, and vertical shift. **3.** The value of c in the function is positive, which means the phase shift is to the left, not to the right. The phase shift is left $\frac{\pi}{4}$ units.
5. amplitude: 4; period: 2π; phase shift: right $\frac{\pi}{6}$ units; vertical shift: up 2 units
7. $y = 2 \cos\left(x + \frac{\pi}{2}\right) - 1$
9. $y = \sin\left(x + \frac{2\pi}{3}\right)$ and $y = \cos\left(x + \frac{\pi}{6}\right)$

11. When $c > 0$, the phase shift is left c units. When $c < 0$, the phase shift is right c units. When $d > 0$, the vertical shift is up d units. When $d < 0$, the vertical shift is down d units. **13.** $y = d$
15. Answers may vary. Sample: The y-intercept is always affected by the value of d. When $c \neq 0$, the phase shift means that a, b, and c can also affect the y-intercept by changing the amplitude and period.

17.



19.

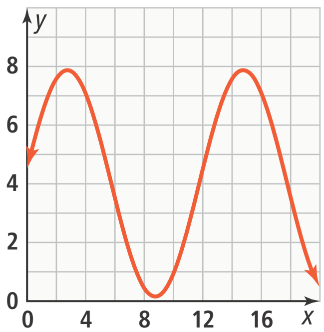


21. amplitude: $\frac{1}{2}$; period: π; phase shift: right $\frac{\pi}{4}$ units; vertical shift: down 1 unit; maximum: $-\frac{1}{2}$; minimum: $-1\frac{1}{2}$
23. $y = 50 \cos\left(\frac{\pi}{15}x\right) + 50$; The average amount of moon visible during the 8 moon phases, 50%, is the same as the midline value, 50.

Selected Answers

Topic 7

25. a. Answers may vary. Sample: $y = 3.855 \cos\left[\frac{\pi}{6}(x - 2.75)\right] + 4.015$
b. The average of the 12 rainfall amounts, about 3.236 inches, is close to the midline value, 4.015 inches.
c.

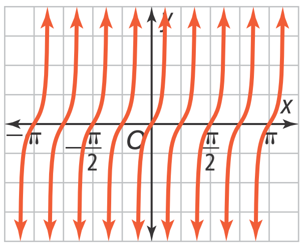


27. B

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** terminal side **5.** frequency **7.** reference angle
9. $\sin\theta = \frac{b}{c}$; $\cos\theta = \frac{a}{c}$; $\tan\theta = \frac{b}{a}$;
$\csc\theta = \frac{c}{b}$; $\sec\theta = \frac{c}{a}$; $\cot\theta = \frac{a}{b}$
11. $\sin\theta = \frac{33}{65}$; $\cos\theta = \frac{56}{65}$; $\tan\theta = \frac{33}{56}$;
$\csc\theta = \frac{65}{33}$; $\sec\theta = \frac{65}{56}$; $\cot\theta = \frac{56}{33}$
13. 6.34 ft **15.** 297° **17.** 194° **19.** 120°
21. −60° or 300° **23.** 30° in Quadrant IV
25. about 1,539 ft
27. $\sin 135° = \frac{\sqrt{2}}{2}$; $\cos 135° = -\frac{\sqrt{2}}{2}$
29. $\sin 420° = \frac{\sqrt{3}}{2}$; $\cos 420° = \frac{1}{2}$

31. $\tan -\frac{\pi}{4} = -1$ **33.** $\sec \frac{8\pi}{3} = -2$; $\csc \frac{8\pi}{3} = \frac{2\sqrt{3}}{3}$; $\cot \frac{8\pi}{3} = -\frac{\sqrt{3}}{3}$ **35.** Answers may vary. Sample: The point $(3\sqrt{2}, -3\sqrt{2})$ represents a position that is about 4.2 miles east and 4.2 miles south of the camp. **37.** amplitude: 3; period: 4π; frequency: $\frac{1}{4\pi}$ **39.** amplitude: 2; period: $\frac{\pi}{3}$; frequency: $\frac{3}{\pi}$ **41.** The graph is compressed vertically which makes the graph look more straight than the parent function $y = \tan x$. The horizontal compression changes the period of the function from π to $\frac{\pi}{2}$.



43. domain: $\{x: x \neq n\pi$ where n is an integer$\}$; range: $(-\infty, \infty)$; period: π; asymptotes: any multiple of π; zeros: $\{x;\ x = \frac{\pi}{2} + n\pi$, where n is an integer$\}$.
45. 6.39 ft **47.** amplitude: $\frac{1}{4}$; period: $\frac{\pi}{3}$; phase shift: $\frac{\pi}{2}$ units to the left; vertical shift: 2 units up

Selected Answers

Topic 8

Lesson 8-1

1. You can use inverse functions and their graphs to find the measures of angles that have a given value of the trigonometric function. **3.** The student wrote the measures of the angles for which the sine is equal to 0. $\sin(0 + \pi x) = \sin(\pi x) = 0$ where x is an integer. The measures of the angles for which the sine is equal to 1 are $\sin\left(\frac{\pi}{2} + 2\pi x\right)$, where x is an integer. **5.** 45° or $\frac{\pi}{4}$ **7.** $\pm 42.27° + (360°)k$, where k is an integer **9.** $\theta = 0.25$ radians and 2.89 radians **11.** There is no angle θ with cosine −1.5 because −1.5 is not an element of the domain of cosine. **13.** The student gave values for $\theta = \cos^{-1}\left(\frac{1}{2}\right)$, rather than sine. The correct answer is $\theta = \frac{\pi}{6} + 2\pi n$ or $\frac{5\pi}{6} + 2\pi n$. **15.** $\sec^{-1}\left(\frac{1}{2}\right)$ does not exist, since $\sec x = \frac{1}{\cos x}$ and $\cos x$ cannot equal 2. **17.** $\sin\theta = \pm 1$ **19.** $0 \le x \le \pi$ **21.** 60° or $\frac{\pi}{3}$ **23.** 120 or $\frac{2\pi}{3}$ **25.** $129.0° + (360°)k$, $231.0° + (360°)k$, where k is an integer **27.** $-32.2° + (180°)k$, $147.8° + (360°)k$, where k is an integer **29.** $\frac{4\pi}{3}, \frac{5\pi}{3}$ **31.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **33. a.** $t = \frac{1}{\pi}\cos^{-1}\left(\frac{h-6}{2}\right)$ **b.** 0.67 s **35.** angle of elevation = 42° **37. a.** no **b.** yes **c.** yes **d.** yes **e.** yes **f.** no **39. Part A** $\theta = 65.4°$ **Part B** $d = 610.5$ ft **Part C** $\ell \approx 254.3$ ft

Lesson 8-2

1. You can use the Law of Sines and the Law of Cosines in non-right triangles to find the measures of sides and angles when you have a side and its corresponding angle as well as another side or angle. **3.** The negative square root is not valid because a is a side length and side lengths cannot be negative. **5.** about 40.3° **7.** about 32.3 **9.** about 10.9 **11.** about 97.6° **13.** Angle-Side-Side **15.** Neither student is incorrect; they reported their answers to different degrees of precision. Since the given values are in integers, 20 in. is probably precise enough. **17.** Inverse sine gives an acute angle, and the supplement to that angle will always be obtuse. This causes a problem when the known angle is acute because both the acute angle and its obtuse supplement could be valid options for the missing angle. However, if the known angle is obtuse, only the acute solution is valid because a triangle cannot contain two obtuse angles. **19.** Draw the altitude from E, label its length x.

$\sin F = \frac{x}{g}$ and $\sin G = \frac{x}{f}$

$g \sin F = x$ and $f \sin G = x$

$g \sin F = f \sin G$. Therefore,

$\frac{\sin F}{f} = \frac{\sin G}{g}$

Selected Answers

Topic 8

21. about 6.9 **23.** two: $m\angle E \approx 61.7°$ or 118.3° **25.** Sample: Student draws altitude outside triangle from vertex L.

$\cos(180° - K) = -\cos K = \frac{x}{j}$

$x = -j \cos K$

$x^2 + h^2 = j^2$ and $(l + x)^2 + h^2 = k^2$

$l^2 + 2lx + x^2 + h^2 = k^2$

$k^2 = l^2 + 2l(-j \cos K) + j^2$

$k^2 = l^2 + j^2 - 2lj(\cos K)$

27. about 96.4° **29.** about 58.7°
31. about 93.6 in. **33.** about 55.6°
35. no; no; no; yes; yes; no
37. Part A 240 ft **Part B** about 55.2° **Part C** about 282.6 ft

Lesson 8-3

1. Relationships that exist between trigonometric functions can be verified and applied through the use of the unit circle and its definitions of sine, cosine, and tangent. **3.** Because the equation $\cos(-x) = \cos x$ is true for all values of the variable for which both sides of the equation are defined. **5.** The value of the tangent is undefined if the cosine is zero. The cosine is zero for $\theta = \frac{\pi}{2} + k\pi$, where k is an integer.

7. $\sec \theta \bullet \cot \theta \stackrel{?}{=} \csc \theta$

$$\frac{1}{\cos \theta} \bullet \frac{\cos \theta}{\sin \theta} \stackrel{?}{=} \csc \theta$$

$$\frac{1}{\cancel{\cos \theta}} \bullet \frac{\cancel{\cos \theta}}{\sin \theta} \stackrel{?}{=} \csc \theta$$

$$\frac{1}{\sin \theta} = \csc \theta \checkmark$$

9. $\tan \theta$ **11.** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **13.** Yes; $\cos 2\theta = \cos(\theta + \theta) = \cos \theta \bullet \cos \theta - \sin \theta \bullet \sin \theta = \cos^2 \theta - \sin^2 \theta$ **15.** The student applied the cosine of sums formula rather than the cosine of differences. The correct answer is $\frac{\sqrt{6} + \sqrt{2}}{4}$.

17. a. $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

b. $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

19. $\sin(90° - x) = \cos x$; $\cos(90° - x) = \sin x$; $\sin 15° = \cos 75°$; $\cos 60° = \sin 30°$
21. $\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos \theta} = \sec \theta$; therefore secant is an even function. **23.** 1

25. $$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

$$= \frac{\sin \alpha \cos \beta + \sin \beta \cos \alpha}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$

$$= \frac{\frac{\sin \alpha \cos \beta + \sin \beta \cos \alpha}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \beta \cos \alpha}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Selected Answers

Topic 8

27. $-2 - \sqrt{3}$; -3.73 **29.** $\frac{-\sqrt{2}}{2}$; -0.707

31. Yes; since the sum of the sound waves is 0 for all values of x, the noises cancel one another.

33. $\left(\frac{5\sqrt{3}}{2}\cos\theta - \frac{5}{2}\sin\theta, \frac{5\sqrt{3}}{2}\sin\theta + \frac{5}{2}\cos\theta\right)$

35.
$$s = \frac{h\sin(90° - \theta)}{\sin\theta}$$
$$= \frac{h\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\theta}$$
$$= \frac{h\cos\theta}{\sin\theta}$$
$$= h\cot\theta$$
37. A

Lesson 8-4

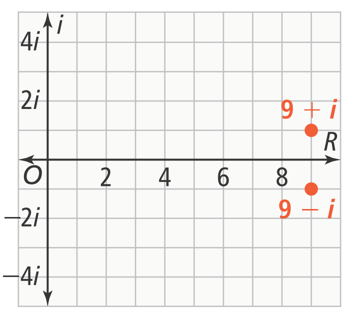
1. To model the complex number $a + bi$, graph the ordered pair (a, b) in the complex plane. To model complex conjugates, use reflection images of their corresponding points over the real axis. To model the modulus of a complex number, find its distance from the origin on the complex plane. To model $r + s$, where r and s are complex numbers, graph the points for r, the origin, and s as three vertices of a parallelogram. The fourth vertex of the parallelogram represents $r + s$. To model $r - s$, graph the points for r, the origin, and the opposite of s as three vertices of a parallelogram. The fourth vertex of the parallelogram represents $r - s$.

3. The complex plane and the standard *xy*-plane both have two axes. Instead of the standard *x*- and *y*-axes, the complex plane has real (horizontal) and imaginary (vertical) axes. **5.** (14, –7) **7.** $\sqrt{10}$ **9.** $6.5 - 2.5i$ **11.** (4, –6) **13.** The student added 9 and $4i$ to get $13i$, but 9 and $4i$ can't be added since they are not like terms.

$$\text{midpoint} = \frac{(5 + 8i) + (4 - 4i)}{2}$$
$$\text{midpoint} = \frac{(5 + 4) + (8 - 4)i}{2}$$
$$\text{midpoint} = \frac{9 + 4i}{2}$$
$$\text{midpoint} = \frac{9}{2} + \frac{4i}{2} = 4.5 + 2i$$

15. Yes, the modulus of $a + bi$ and the modulus of $a - bi$ are both $\sqrt{a^2 + b^2}$.

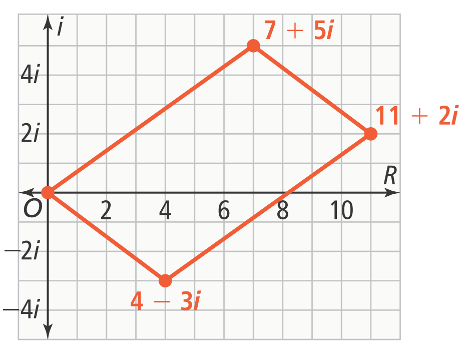
17.



19. (–1, –3) **21.** (–8.5, 0.5)

23. $\sqrt{82} \approx 9.06$

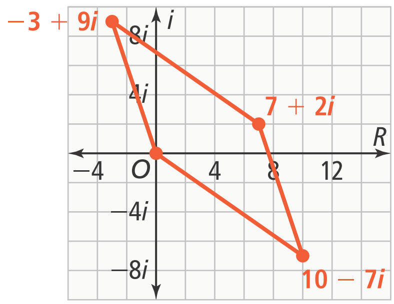
25.



Selected Answers

Topic 8

27.

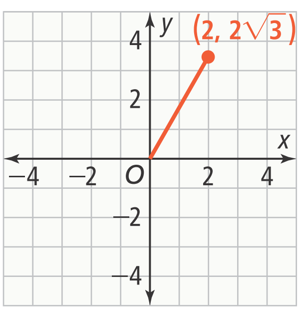


29. $\sqrt{101} \approx 10.05$ **31.** $\sqrt{\frac{45}{32}} \approx 1.19$ units **33.** (9, –14) **35.** C

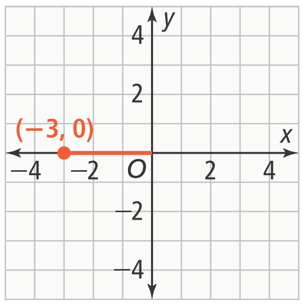
Lesson 8-5

1. You can use trigonometric functions to convert the complex numbers to both rectangular and polar form. Then you can represent and operate with the numbers in these forms.
3. The argument of a complex number is the measure of the angle θ between the segment and the real axis.
5. 4 cis $\frac{\pi}{6}$ **7.** –5 **9.** 12 cis π
11. 6 cis $\frac{\pi}{2}$ **13.** $-112 - 384i$ **15.** $-7 + 24i$; the results are the same; student preferences will vary **17.** The student found the argument θ to be $\frac{5\pi}{6}$, when it should have been $\frac{11\pi}{6}$, so the answer should be 2cis$\left(\frac{11\pi}{6}\right)$. **19.** $-\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

21.



23.



25. $2i$ **27.** $2\sqrt{3} - 2i$ **29.** 2 cis $\frac{5\pi}{6}$
31. 6.4 cis 0.9 **33.** 6 cis $\frac{11\pi}{6}$; $3\sqrt{3} - 3i$
35. $z_1 = 2\sqrt{2}$ cis $\frac{7\pi}{4}$; $z_2 = 3\sqrt{2}$ cis $\frac{3\pi}{4}$; $z_1z_2 = 12$ cis $\frac{5\pi}{2}$; $12i$ **37.** $16\sqrt{2}$ cis $\frac{9\pi}{4}$; $16 + 16i$ **39.** 6 cis $\frac{13\pi}{6} = 3\sqrt{3} + 3i$ volts
41. Answers may vary. Sample: $r = 5\cos(9\theta)$. It appears as through when n is even, the number of petals is $2n$ and when n is odd, the number of petals is n. **43.** A

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** complex plane
5. imaginary axis **7.** argument
9. 45° or $\frac{\pi}{4}$ **11.** 120° or $\frac{2\pi}{3}$ **13.** $\frac{3\pi}{4}$, $\frac{5\pi}{4}$
15. 51° **17.** 8.5 **19.** 92.9° **21.** 43.8°
23. sec x csc x **25.** $\frac{1 - \cos x}{1 + \cos x}$
27. $\frac{\sqrt{2} - \sqrt{6}}{4}$ **29.** $\frac{\sqrt{6} + \sqrt{2}}{4}$
31. No; the sum of the sound waves is $2\sin(1{,}500\pi x)$. **33.** (6, 4i) **35.** $\sqrt{10}$
37. $\sqrt{941}$ **39.** (1, i) **41.** $-3 + 3\sqrt{3}i$
43. 2.24 cis 4.25 **45.** $46 - 9i$ **47.** 6 cis $\frac{7\pi}{12}$

Selected Answers

Topic 9

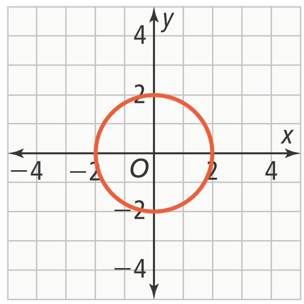
Lesson 9-1

1. A parabola is the set of all points for which the distance between the point and the focus point and the distance from the point and the directrix line are equal. Algebraically, this results in an equation in degree 2 for either x or y, and in degree 1 for the other. **3.** The parabola $y^2 = -9x$ has the x-axis as its line of symmetry, and it opens to the left. **5.** It is measured by the perpendicular from the point to the line. **7.** $x = -\frac{1}{4}y^2$ **9.** focus: (3, –0.75); directrix: $y = -1.25$ **11.** the way in which the plane intersects the double-right cone **13.** The parabola will appear very long and narrow. **15. a.** focus: (–3, –4.75); directrix: $y = 5.25$ **b.** $x = -3$ **c.** (–7, 11) **17.** $x = y^2$ **19.** $y = -\frac{1}{32}x^2$ **21. a.** vertex (1, 3); focus $\left(\frac{9}{8}, 3\right)$; directrix: $x = \frac{7}{8}$ **b.** vertex (4, 0) focus (4, –6); directrix $y = 6$ **23.** $y = \frac{1}{12}x^2$ **25.** C, D **27. Part A** $x = \frac{1}{4}y^2$ **Part B** $\frac{9}{4}$ cm deep

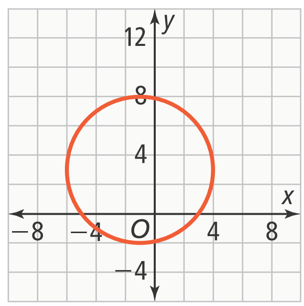
Lesson 9-2

1. A circle is a round plane figure that has a circumference with points equidistant from the center. The formula $(x - h)^2 + (y - k)^2 = r^2$ gives the center (h, k) and the radius r. **3.** In the equation $r^2 = 6$, Cole squared 6 to find the radius instead of taking the square root of 6 to find the radius. The length of the radius is $\sqrt{6}$. **5.** (0, 0); 5 **7.** (1, –6); $\sqrt{5}$ **9.** $x^2 + y^2 = 64$ **11.** $(x - 2)^2 + (y + 3)^2 = 25$ **13.** The circle is translated 2 units left and 2 units up, not 2 units right and 2 units down. **15. a.** (6, 0) and (–6, 0) **b.** (0, 6) and (0, –6) **17.** No; a circle cannot have a radius of 0.

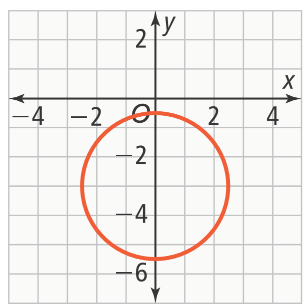
19. $x^2 + y^2 = 4$;



21. $(x + 1)^2 + (y - 3)^2 = 25$;



23. $x^2 + (y + 3)^2 = 7$;



25. $(x - 3)^2 + (y - 5)^2 = 100$
27. The equation of this circle in standard form is $(x + 5)^2 + (y - 1)^2 = 25$. The circle has center (–5, 1) and radius 5.
29. The equation of this circle in standard form is $(x - 2)^2 + (y - 4)^2 = 25$. The circle has center (2, 4) and radius 5.

Selected Answers

Topic 9

31. (2, 3) and (–2, –3) **33.** (4, –2) and (–4, 2) **35. a.** $x^2 + y^2 = 36$ **b.** $(-3\sqrt{(3)}, -3)$ and $(3\sqrt{(3)}, -3)$ **37.** Write the equation of the circle given by the equation $x^2 + y^2 + 2x - 14y - 6 = 8$ in standard form. Identify the center and radius. Then sketch the graph. Standard form: $(x - \underline{-1})^2 + (y - \underline{7})^2 = \underline{64}$
Center: ($\underline{-1}$, $\underline{7}$) Radius: $\underline{8}$
39. Part A Circle 1: $x^2 + y^2 = 16$;
Circle 2: $(x - 1)^2 + (y + 2)^2 = 25$;
Circle 3: $(x + 3)^2 + (y - 6)^2 = 36$;
Circle 4: $(x + 4)^2 + (y + 7)^2 = 256$
Part B $(x - h)^2 + (y - k)^2 = \left(\frac{d}{2}\right)^2$

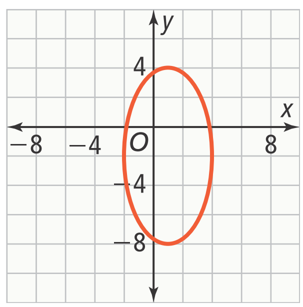
Lesson 9-3

1. The equation in standard form of an ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. The coordinates (h, k) give the center of the ellipse. The length of the horizontal axis is $2a$ and the length of the vertical axis is $2b$. When $a > b$, the horizontal axis is the major axis, the vertices are located at $(h \pm a, k)$, the co-vertices are located at $(h, k \pm b)$, and the foci are located at $(h \pm c, k)$, where $a^2 = c^2 + b^2$. When $a < b$, the vertical axis is the major axis, the vertices are located at $(h, k \pm b)$, the co-vertices are located at $(h \pm a, k)$, and the foci are located at $(h, k \pm c)$, where $b^2 = c^2 + a^2$. **3.** Darren found the coordinates of the co-vertices; the vertices are at (9, –1) and (–1, –1).
5. vertices: (0, 8) and (0, –8); co-vertices: (7, 0) and (–7, 0)
7. $(0, \sqrt{6})$ and $(0, -\sqrt{6})$, or approximately (0, 2.4) and (0, –2.4)

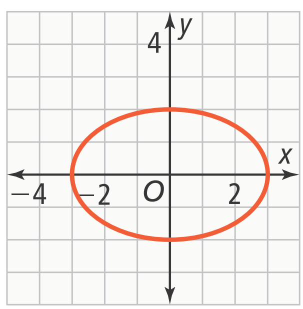
9. When the values of a and b are equal, the vertices and co-vertices will be the same distance from the center of the ellipse, which forms a circle.
11. Kendall incorrectly thought the ellipse was centered at the origin. The vertices are located at (4, –1) and (4, –11). The co-vertices are located at (7, –6) and (1, –6). The foci are located at (4, –2) and (4, –10). **13.** When c is close to 0, the values of a and b are close to the same. This means the area of the ellipse can be written as $A = \pi(a)(a)$ or $A = \pi a^2$. This formula is the same as the area of a circle with a radius of a.

15. $\frac{x^2}{36} + \frac{y^2}{27} = 1$

17.



19.

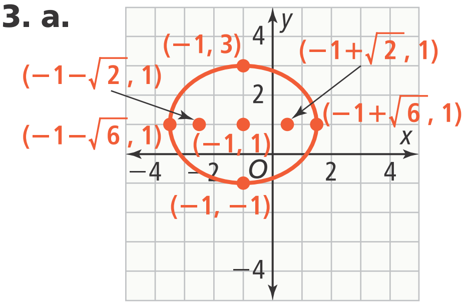


21. $\frac{x^2}{73} + \frac{y^2}{9} = 1$

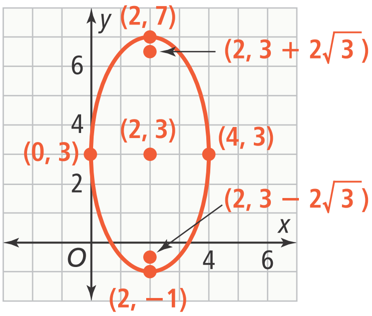
Selected Answers

Topic 9

23. a.



b.



25. $\frac{(x-2)^2}{36} + \frac{(y+2)^2}{64} = 1$ **27.** A, C, E

29. Part A 88.7 in. or 7.4 ft
Part B 430 in. or 35.8 ft

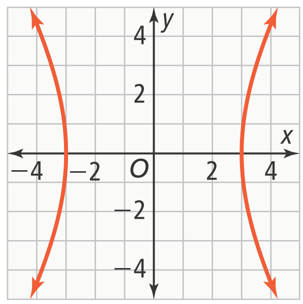
Lesson 9-4

1. For a horizontal hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the vertices are located at ($\pm a$, 0), the asymptotes are located at $y = \pm\frac{b}{a}x$, and the foci are located at ($\pm c$, 0), where $c = \sqrt{a^2 + b^2}$. **3.** The line that goes from one vertex to the other vertex, through the center, is the transverse axis. **5.** Write the second-degree equation in general form $Ax^2 + Cy^2 + Dx + Ey + F = 0$. If A or C (but not both) is zero, then the equation represents a parabola. If the A and C are equal, then the equation represents a circle. If A and C are both positive, but different, then the equation represents an ellipse. If A and C have opposite signs, then the equation represents a hyperbola.

7. (11, 0) and (–11, 0) **9.** (0, 5) and (0, –5) **11.** $y = \pm\frac{1}{3}x$

13. a. The coefficients A and C in the equation have opposite signs, so it is a hyperbola. **b.** Divide each side of the equation by 225; $\frac{x^2}{9} - \frac{y^2}{25} = 1$ **c.** (–3, 0) and (3, 0), **d.** $y = \pm\frac{5}{3}x$ **e.** ($-\sqrt{34}$, 0) and ($\sqrt{34}$, 0) **f.** The transverse axis is horizontal because the vertices and foci lie along the x-axis.

g.



15. The equation shows that there is a vertical transverse axis, so the vertices are at (0, –12) and (0, 12), not (–12, 0) and (12, 0). Also, because the transverse axis is vertical, the asymptotes are at $y = \pm\frac{3}{4}x$, not $y = \pm\frac{4}{3}x$. Finally, $c^2 = 144 + 256 = 400$, so $c = 20$. This means the foci are at (0, –20) and (0, 20), not (0, –400) and (0, 400).

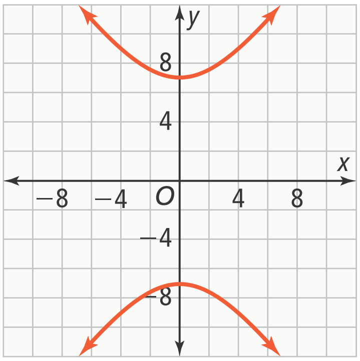
17. The equation $\frac{x^2}{18} - \frac{y^2}{32} = -2$ is a hyperbola because after dividing both sides of the equation by – 2, the result is $-\frac{x^2}{36} + \frac{y^2}{64} = 1$. Then the x^2- and y^2-terms can be switched using the Commutative Property of Addition. The result is $\frac{y^2}{64} - \frac{x^2}{36} = 1$, which is in the standard form of the equation of a hyperbola.

19. $\frac{y^2}{81} - \frac{x^2}{144} = 1$

Selected Answers

Topic 9

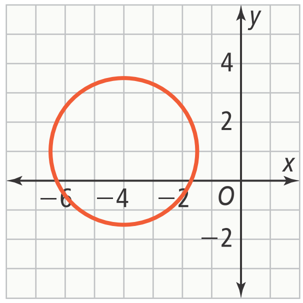
23.



25. $\frac{x^2}{81} - \frac{y^2}{144} = 1$ **27.** ellipse
29. parabola
33. $\frac{x^2}{6{,}400{,}000{,}000} - \frac{y^2}{22{,}500{,}000{,}000} = 1$; 20,089 km **35.** E

Topic Review

1. Check students' work. See *Teacher's Edition* for details **3.** center **5.** conic section **7.** $y = \frac{1}{4}x^2$ **9.** $x = -\frac{1}{8}y^2$
11. $y = \frac{1}{3}x^2$
13. $(x + 4)^2 + (y - 1)^2 = 6.25$



15. $(x - 2)^2 + (y - 2.4)^2 = 1.81$
17. $\frac{y^2}{9} - \frac{x^2}{576} = 1$ **19.** $\frac{x^2}{1{,}600} + \frac{y^2}{700} = 1$; 26.5 ft

Selected Answers

Topic 10

Lesson 10-1

1. Each element in a matrix is in a particular row and a particular column. Use the meaning of each row and each column to interpret the element. Adding and subtracting with matrices involves adding and subtracting pairs of corresponding elements. Scalar multiplication involves multiplying all elements by a single number. The result of addition, subtraction, and scalar multiplication is a matrix with the same dimensions. **3.** The two matrices must have the same dimensions. If they have the same dimensions, you can add corresponding elements. **5.** 5

7. $\begin{bmatrix} 3 & 5 \\ 3 & 13 \end{bmatrix}$ **9.** $\begin{bmatrix} 12 & -8 \\ 28 & 4 \end{bmatrix}$ **11.** $\begin{bmatrix} 6 & 14 \\ 2 & 10 \end{bmatrix}$

13. $\begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix}$ **15.** To find $A + B$, you would add corresponding elements. To find $A - B$, you would subtract corresponding elements. To find C, find the additive inverse of A.

17. If the sum of two matrices is the zero matrix, then the two matrices are additive inverse matrices.

19. $d_{22} = 6$ means 6 senior girls; $d_{12} = 5$ means 5 junior girls; $d_{11} = 4$ – 4 junior boys

21. not possible **23.** $\begin{bmatrix} 7 & 5 & 8 \\ 5 & -5 & 12 \end{bmatrix}$

25. $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$ **27.** $[-4 \quad 7 \quad 8 \quad -9]$

29. $Y(10, -2)$; $Z(11, 10)$ **31.** Check students' work.

33. $C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

35. C

Lesson 10-2

1. To multiply two matrices means to multiply all of their corresponding columns and rows. **3.** Because it is similar to the Identity Property of Multiplication; when you multiply a matrix A by 1 (or in this case the identity matrix), the product is the matrix A itself.

5. $AB = \begin{bmatrix} 3 & 0 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix} =$

$$\begin{bmatrix} 3(-2) + 0(3) & 3(1)\ 0(-4) \\ -1(-2) + -2(3) & -1(1) + -\ 2(-4) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 3 \\ -4 & 7 \end{bmatrix}$$

$BA = \begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & -2 \end{bmatrix} =$

$$\begin{bmatrix} -2(3) + 1(-1) & -2(0) + 1(-2) \\ 3(3) + -4(-1) & 3(0) + -4(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -2 \\ 13 & 8 \end{bmatrix}$$

7. $\begin{bmatrix} 5 & 0 \\ 8 & 2 \end{bmatrix}$ **9.** The product also has dimensions $n \times n$. **11.** The student multiplied each pair of corresponding values instead of multiplying the first row by the first column, and so on. The correct product is $\begin{bmatrix} 2 & -4 \\ 23 & -10 \end{bmatrix}$.

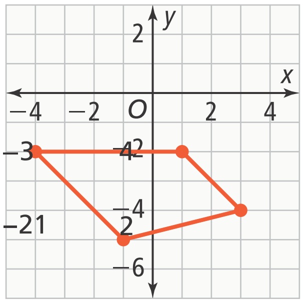
13. $F = [81.5 \quad 87.5]$ **15.** Yes; they both equal $\begin{bmatrix} 12 & -18 \\ -32 & 14 \end{bmatrix}$.

Selected Answers

Topic 10

17. a. $A = \begin{bmatrix} 1 & -4 & 3 & -1 \\ -2 & -2 & -4 & -5 \end{bmatrix}$

b.



The graph is reflected across the *x*-axis and then rotated 90° clockwise.

19. $\begin{bmatrix} 45 & 20 \\ 50 & 15 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 25 \end{bmatrix} = \begin{bmatrix} 1400 \\ 1375 \end{bmatrix}$

The baseball park stand made \$1,400 in sales, and the football stadium stand made \$1,375 in sales.

21. $\begin{bmatrix} -3 & 4 \\ -21 & 2 \end{bmatrix}$

23. Part A $C = \begin{bmatrix} 0.50 \\ 1 \\ 5 \\ 7 \end{bmatrix}$; $P = \begin{bmatrix} 1 \\ 2 \\ 12 \\ 15 \end{bmatrix}$;

$S = \begin{bmatrix} 20 & 15 & 40 & 30 \\ 25 & 20 & 50 & 35 \\ 15 & 20 & 60 & 45 \end{bmatrix}$

Part B $X = P - C$; $X = \begin{bmatrix} 0.50 \\ 1 \\ 7 \\ 8 \end{bmatrix}$

Part C $SX = \begin{bmatrix} 545 \\ 662.5 \\ 807.5 \end{bmatrix}$

Each element represents Paula's business profits for each of the three years.

Lesson 10-3

1. To perform arithmetic operations on vectors, you perform the operations to each of the components (the magnitude and direction) of the vector. **3.** You add the opposite of the vector. **5.** Quadrant II **7.** $\langle -12, 1 \rangle$ **9.** $\langle -3.4, -4.9 \rangle$ **11.** You can use the Distance Formula or the Pythagorean Theorem. **13.** Because the magnitude is a distance measure, it must be positive. **15.** $\langle 2, -6 \rangle$ **17.** $\langle -5, -2 \rangle$; magnitude $= \sqrt{29} \approx 5.4$; direction $\approx 201.8°$ **19.** $\langle 7, 3 \rangle$; magnitude $= \sqrt{58} \approx 7.6$; direction $\approx 23.2°$ **21.** $\langle 7, 4 \rangle$ **23.** $\langle 7, 1 \rangle$ **25.** $\langle 14, 4 \rangle$; magnitude ≈ 14.6; direction $\approx 16.0°$ **27.** $\langle -3, 7 \rangle$; magnitude ≈ 7.6; direction $\approx 113.2°$ **29.** $\langle -24, -72 \rangle$ **31.** Multiply $\overrightarrow{EF} = \langle 8, 4 \rangle$ by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ to get $\langle -8, 4 \rangle$. $\langle -8, 4 \rangle$ is the image of $\overrightarrow{EF}$ after reflection across the *y*-axis. **33. a.** $\langle 40, 600 \rangle$ **b.** 40 represents the wind speed of 40 mph; 600 represents the plane speed of 600 mph. **c.** 601 mph represents the speed of the plane relative to the ground. **d.** 3.8° **35.** B, D **37. Part A** $\langle 3, 3\sqrt{3} \rangle$ **Part B** $\langle 5, 3\sqrt{3} \rangle$ **Part C** 7.2 mi

Lesson 10-4

1. The multiplicative inverse of a square matrix A is the unique matrix A^{-1} such that $A \cdot A^{-1} = I$ and $A^{-1} \cdot A = I$. For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse matrix is $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. If A is an $n \times n$ matrix with $n \geq 3$, use technology to calculate the inverse matrix.

Selected Answers

Topic 10

3. The value of the determinant is $3(8) - 6(4) = 0$. Because the determinant is equal to 0, the matrix does not have an inverse.

5. $\begin{bmatrix} 1.5 & 2 \\ -1 & -1 \end{bmatrix}$ **7.** $\begin{bmatrix} -\frac{1}{6} & 0 & \frac{1}{12} \\ -\frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ -\frac{11}{6} & -1 & -\frac{1}{12} \end{bmatrix}$

9. 10 square units **11.** Sample: $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$; The value of the determinant is 0, so the matrix does not have an inverse.
13. Leah found the area of a triangle with one vertex at the origin, defined by the vectors $\langle 3, 6 \rangle$ and $\langle -4, -10 \rangle$. The area of the parallelogram is $|\det T| = |-6| = 6$ square units.
15. $a = \pm 1$, $b = 0$, $c = 0$, $d = \pm 1$
17. $b = -2$, when $b = -2$, the determinant of the matrix is 0. If $\det A = 0$, the matrix has no inverse.
19. $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ **21.** $R^{-1} = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$
23. $S^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$ **25.** DETERMINANTS
27. $13\frac{1}{2}$ square units **29.** 16 square units
31. a. $\begin{bmatrix} 20 & 50 \\ 80 & 10 \end{bmatrix}$ **b.** 1,900 square feet
c. \$125.86; divide to find the number of bags of grass seed that will cover the area of the park: $1{,}900 \div 300 \approx 6.3$. Since partial bags of grass seed cannot be purchased, the landscaping company will need to purchase 7 bags of grass seed: $\$17.98 \times 7 = \$125.86$.
33. a. yes **b.** yes **c.** yes **d.** no
35. Part A 9999 4630 1057 2258
Part B Check students' work.
Part C The credit card number would have to be in a matrix that has 3 rows. The matrix would have to have 6 columns to allow for the 16 digits, but there would be 2 blank spaces. A designation of a specified number, such as –1, would have to be made and used to represent the lack of a digit in that place.

Lesson 10-5

1. Express the linear system of equations as a matrix equation. Then, find the determinant of the coefficient matrix, A. If $\det A \neq 0$, solve the matrix equation using the inverse of the coefficient matrix. If $\det A = 0$, solve the original system with an alternative method. Use substitution, elimination, or matrix row reduction. **3.** Write all the equations in standard form. Create a matrix with the same number of rows as equations and the same number of columns as variables. For each position in the matrix, enter the coefficient of the corresponding variable in the corresponding equation, including a zero for any variable that does not appear in a particular equation.
5. $\begin{bmatrix} 5 & 3 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -21 \\ -24 \end{bmatrix}$
7. $A^{-1} \begin{bmatrix} \frac{2}{7} & \frac{27}{70} & \frac{1}{70} \\ -\frac{1}{7} & -\frac{17}{70} & -\frac{11}{70} \\ -\frac{2}{7} & -\frac{3}{35} & -\frac{4}{35} \end{bmatrix}$; $x = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$
9. Matrix multiplication is not commutative. To get X on the left side of the equation, you have to multiply by A^{-1} on the left, so you must multiply both sides of the equation by A^{-1} on the left.

Selected Answers

Topic 10

11. The student incorrectly multiplied both sides of the matrix equation by $\begin{bmatrix} 5 & 9 \\ 2 & -3 \end{bmatrix}$ instead of multiplying both sides of the matrix equation by $\begin{bmatrix} 5 & 9 \\ 2 & -3 \end{bmatrix}^{-1}$. The correct solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$.

13. Answers will vary. Sample:

$$\begin{aligned} 2w + x - 3y + 4z &= -13 \\ w + x - y - z &= 0 \\ 4w + 3x - 2z &= 5 \\ 6x - 4y + 5z &= -24 \end{aligned};$$

$$\begin{bmatrix} 2 & 1 & -3 & 4 \\ 1 & 1 & -1 & -1 \\ 4 & 3 & 0 & -2 \\ 0 & 6 & -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \\ 5 \\ -24 \end{bmatrix};$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}$$

15. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ -2 \end{bmatrix}$

17. $\begin{bmatrix} 2 & 3 & 7 \\ 10 & 8 & -2 \\ 6 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -12 \\ 30 \end{bmatrix}$

19. $x = 0$, $y = -7$, $z = 5$

21. $x = -2$, $y = 3$, $z = -6$

23. a. $\begin{bmatrix} 0.25 & 0.1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2.55 \\ 0 \end{bmatrix}$
b. 9 quarters and 3 dimes

25. 2 servings of cereal A, 4 servings of cereal B, 5 servings of cereal C **27.** D

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** identity matrix **5.** magnitude **7.** coefficient matrix
9. $\begin{bmatrix} -16 & 6 \\ 5 & -4 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ -7 & -7 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -10 & 3 \end{bmatrix}$, $Y(8, -10)$, $Z(5, -3)$

13. $\begin{bmatrix} 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \end{bmatrix} = \begin{bmatrix} 24 \\ 20 \end{bmatrix}$; 24 women, 20 men **15.** $\begin{bmatrix} 18 & 2 \\ 18 & -18 \end{bmatrix}$ **17.** $\begin{bmatrix} -28 & 93 \\ -20 & 24 \end{bmatrix}$

19. $\begin{bmatrix} -37 & 47 \\ -4 & 14 \end{bmatrix}$ **21.** Matrices can be multiplied when the number of columns in the first matrix is equal to the number of rows in the second matrix. **23.** $\langle 5, 12 \rangle$; $\langle 11, 8 \rangle$ **25.** $\langle 52, -12 \rangle$ **27.** 38.7° **29.** 6

31. $\begin{bmatrix} -\frac{10}{7} & -\frac{3}{2} & \frac{4}{7} \\ \frac{5}{7} & \frac{1}{2} & -\frac{2}{7} \\ \frac{1}{7} & 0 & \frac{1}{7} \end{bmatrix}$ **33.** 22 square units

35. $x = -9$, $y = -7$, $z = 3$ **37.** \$1 for each folder and \$5 for each notebook

Selected Answers

Topic 11

Lesson 11-1

1. Statistics can be used to study variables among entire populations, or groups (samples) within the populations, that can pertain to qualities or quantities. **3.** A categorical variable (also referred to as a qualitative variable) is used to study data that can be classified into groups, whereas a quantitative variable is a numeric data value that can be counted, added, subtracted, and so on. **5.** The question is not a statistical question because it can be answered using one piece of information. **7.** Statistic; the data were collected from a sample, not the entire population. **9.** This is a parameter because it involves the entire population of the restaurant's customers during that month and because the restaurant wanted to know only about that one month. **11. a.** Sample: What is your favorite primary color? **b.** Sample: What is your hourly wage? **13.** The statistical variable is categorical because the values belong to a limited set of possible qualitative responses. **15.** The data summary shown in the histogram represents a parameter because it shows all the numbers of participants within different age brackets in a 5K. **17.** No; this is a response to the question "What is the recommended amount of Vitamin C people of varying ages should consume per day?" This is a question that is answered with one predetermined piece of information. **19. a.** The sample is the students whose names were randomly drawn. **b.** The population is all of Ms. Lee's students. **21.** C

Lesson 11-2

1. Try to conduct a study where objects are selected randomly, and without bias. **3.** Water sampled from only one pond cannot possibly reflect all pond water in the city. This was an example of convenience sampling. **5.** a self-selected sample **7. a.** No; it would be biased because they will ultimately benefit from the increased traffic flow. **b.** No; it would be biased because they will welcome neither the disruption of caused by the construction nor the increased traffic flow. **c.** This is unbiased, and each homeowner within the city has an equal chance of being selected for the sample. **9.** While it is a systematic sampling method, it is biased towards dogs because all the members of the sample are ticket holders to a dog show. **11. a.** observational **b.** sample survey **13. a.** This is a stratified random sample. It is unbiased as long as the samples chosen within each unit are chosen randomly. **b.** This is a cluster sample. It is unbiased as long as the slips of paper were selected at random. **15.** The bias is in the question. The experimenters only provided those surveyed with a benefit of offshore drilling; however, they failed to inform the participants of the costs. **17.** observational study

Selected Answers

Topic 11

19. Calling between the hours of 5:00 and 7:00 may exclude households with people who work during that time. Calling numbers from a phone registry will exclude households that are not listed. **21.** B

Lesson 11-3

1. Look at the shape of a data distribution to determine if it is skewed left, skewed right, or symmetrically distributed. Use mean and standard deviation to determine center and spread for symmetric distributions, and use median and interquartile range to determine center and spread for skewed distributions. **3.** When the mean is approximately equal to the median, the data set is more likely to be symmetric. This means the mean should be used to describe the center and the standard deviation should be used to describe the spread.
5. mean: 8; standard deviation: ≈ 2.97; minimum: 3; 1st quartile: 5.5; median: 8; 3rd quartile: 9.5; maximum: 14
7. The data are symmetric and approximately normally distributed with a mean of about 5. **9.** Sample: The probability of drawing each of 6 colored marbles from a hat, replacing the marble for each draw.
11. a. approximate values for the five-number summary: minimum: 80; first quartile: 82; median: 85.5; third quartile: 93; and maximum: 100

b. There are 7 data values below the median and 7 data values above the median. The median is 85.5, so the 7th and 8th data values could be 85 and 86. The minimum value is 80 and the first quartile is 82. So, the 1st data value is 80 and the 4th data value could be 82. There are two data values greater than or equal to the minimum value and less than or equal to the first quartile. So, the 2nd and 3rd data values could be 81 and 81. There are two data values greater than or equal to the first quartile and less than or equal to the median. So, the 5th and 6th data values could be 83 and 85. The third quartile is 93 and the maximum value is 100. So, the 11th data value could be 93 and the 14th data value is 100. There are two data values greater than or equal to the median and less than or equal to the third quartile. So, the 9th and 10th data values could be 91 and 92. There are two data values greater than or equal to the third quartile and less than or equal to the maximum value. So, the 12th and 13th data values could be 96 and 98. Therefore, a possible data set is 80, 81, 81, 82, 83, 85, 85, 86, 90, 92, 93, 96, 98, 100. **13.** mean: 30.4; standard deviation: 10.1; minimum: 12; first quartile: 25; median: 33; third quartile: 35; and maximum: 48
15. Normal distribution, so the mean and standard deviation best represent the data set; mean: 43.1; standard deviation: 15.3 **17.** Normal distribution, so the mean and standard deviation best represent the set; mean: 8; standard deviation: 4.1 **19.** skewed right **21.** normally distributed

Selected Answers

Topic 11

23. normal distribution; mean = 18; standard deviation = 5.3
25. skewed left; median = 47.5; interquartile range = 21 **27. a.** The distribution is skewed right with a large outlier. **b.** center = median: \$67,500; spread = interquartile range: \$42,500; Use these because the distribution is skewed.
29. no; yes; yes; no
31. Part A Both graphs show a distribution that is skewed right because most of the data are on the left and the long tail is to the right, so the median age will likely be less than the mean age. **Part B** The second graph would be the best to convince students to continue their lessons into their 20s, because it makes it look like there is a steep increase in contestants starting at age 20 and dropping off at age 30. The first graph would be the best to convince students to continue lessons into their 30s because it makes it look like the number of contestants does not drop off sharply until age 35.

Lesson 11-4

1. The area under the normal distribution curve tells what percent of the data is one, two, three, or more standard deviations away from the mean. **3.** You can use your calculator or a spreadsheet to find the area under the curve to the left of any *z*-score in the standard normal distribution, but it is more complicated to find the area under the curve to the left of a data value in a general normal distribution. **5.** 95%
7. Determine how many standard deviations away from the mean the endpoints of the interval are, then use the corresponding percentage from the Empirical Rule.
9. Answers may vary. Sample: To find the *z*-score, use the formula

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}},$$

$$\text{not } z = \frac{\text{mean} - \text{data value}}{\text{standard deviation}}.$$

The *z*-score is –1.5.
11. The mean is the value at the peak of the graph, or 46.5. Subtract the value of one standard deviation above the mean, where the curvature of the graph changes, from the mean: 55 – 46.5 = 8.5. So, the standard deviation is 8.5. **13.** between 35,000 miles and 65,000 miles
15. 99.7% **17.** 16% **19.** 0.9842, or 98.42% **21.** 0.2676, or 26.76%
23. 0.9177, or 91.77% **25.** 1.5 **27.** –2
29. a. between 20 min and 40 min **b.** less than 10 min **c.** 30.85%
31. Anna's *z*-score = $\frac{89-68}{10} = 2.1$;
Damian's *z*-score = $\frac{95-76}{12} \approx 1.6$; Anna's score is better relative to her exam than Damian's. Although her raw score is lower, she scored more than 2 standard deviations above the mean. **33.** D

Lesson 11-5

1. Calculate the margin of error, which creates a range of reasonable values used to estimate the population parameter based on a sample statistic.
3. A sampling distribution is the distribution of sample statistics such as means or proportions from different samples of the same population.

Selected Answers

Topic 11

5. 11%; ±4% **7.** ±1.9 **9.** 87%
11. 625; Substitute 0.04 for *margin of error* in the formula Margin of Error $= \frac{1}{\sqrt{n}}$: $0.04 = \frac{1}{\sqrt{n}}$. Multiply both sides by $\sqrt{n}$: $0.04\sqrt{n} = 1$. Divide both sides by 0.04: $\sqrt{n} = 25$. Square both sides: $n = 625$. **13.** Sample E; A smaller sample size has more variability and a greater standard deviation.
15. 7 samples had an average time to get ready for school between 11 and 20 minutes. The parameter, average time to get ready for school, is likely to fall between 11 and 50 minutes. (Answers may vary slightly. Accept all reasonable responses.) **17.** About 95% of random samples with 100 test scores will have mean values within $\frac{2(12)}{\sqrt{100}} = 2.4$ points of the population parameter, 72. The range of reasonable means is between 69.6 and 74.4. Since Hailey's sample mean is above the range of the reasonable means, she can conclude that students from her school scored better than the state average.
19. a. Sample: 1–40 **b.** Sample: 41–100 **c.** 36%; This one simulation does not refute Mike's claim, as 36% is relatively close to the observed percentage of 34%. **21. a.** 77% **b.** ±13% **c.** 64%; 90% **23. Part A** Check students' work. **Part B** Check students' work. **Part C** Check students' work.

Lesson 11-6

1. You write an alternative hypothesis to state what you think or might want to show to be true. A null hypothesis is basically the opposite, that the alternative hypothesis is not true. You decide which hypothesis the data supports by sampling, creating random data simulations, and examining the evidence to see which hypothesis it supports. **3.** The null hypothesis states there is no difference between the parameter and the benchmark, whereas the alternative hypothesis states that there is a difference between them. **5.** Null hypothesis: The work with the batting coach did not improve the player's batting average, H_0: $b \leq 0.278$. Alternative hypothesis: The work with the batting coach improved the player's batting average, H_a: $b > 0.278$. **7.** The means for the original samples are 10.42 for TomTom and 12.1 for Hugemato, with a difference of 1.68. Answers may vary; Sample: After randomization, one group is {12.1, 11, 10.4, 12.9, 10.1} and the other is {13.1, 11.2, 10.5, 9.9, 11.4}. The means are 11.3 and 11.22 with a difference of 0.08. This difference is much smaller and suggests that Hugemato tomatoes may actually be larger. **9.** The number 230 should have been used instead of 250 in both the null and alternative hypotheses.

Selected Answers

Topic 11

11. a. The process of randomizing samples, or rearranging data values in random order, allows you to estimate the difference between groups that arises due to natural variability. Then you can decide whether the data gives you enough evidence to support your hypothesis. **b.** The red vertical line is in an area of the distribution with many data values. Many samples have a larger difference between means, so the difference in the means of the original sample groups is likely to be due solely to natural variation.
13. a. sample mean without vitamins: $\bar{x} = 16.6$; sample mean with vitamins: $\bar{x} = 16.2$; The difference in the sample means is $16.2 - 16.6 = -0.4$.
b. 18, 19, 16, 16, 14 **c.** first sample group: $\bar{x} = 16.2$; second sample group: $\bar{x} = 16.6$; The difference in the sample means is $16.6 - 16.2 = 0.4$. **15.** $m = 0.12$
17. a. sample mean new group 1: $\bar{x} = 45.0$; sample mean new group 2: $\bar{x} = 44.8$; The difference in the sample means is $45 - 44.8 = 0.2$. **b.** The results of the randomization do not provide evidence that the difference in the original two sample means is due to the effects of the fertilizer. Since the difference between the two random groups is as great as the difference between the two original groups, the original difference could be due to chance. **19.** C, D **21. Part A** H_0: $\mu = 16$ ounces; H_a: $\mu < 16$ ounces
Part B $m = 0.07$
Part C $15.53 \leq \mu \leq 15.67$
Part D The mean weight of 16 ounces claimed by the bread company is outside the range of reasonable means, so the evidence does not support their claim.

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** experiment
5. statistical question **7.** no
9. Answers may vary. Sample: A parameter is a measure that applies to an entire population. A statistic is a measure that applies to a sample of the population. **11.** observational study **13.** An experimenter imposes a treatment on one group, called the experimental group, but not on the other group, called the control group. **15.** mean: 27.2; standard deviation: 6.3; minimum: 18; first quartile: 23; median: 26.5; third quartile: 33; and maximum: 38
17. measure of center: mean; measure of spread: standard deviation
19. Answers may vary. Sample: the age of children at a day care center
21. 0.6141, or 61.41% **23.** 0.8133, or 81.33% **25.** 0.7486, or 74.86%
27. between 61 and 85 beats per minute **29.** 0.76, or 76% **31.** ± 3.5
33. Answers may vary. Sample: range: 14% to 27% **35.** The original difference of 2.8 points for the sample means of the exam scores is very unusual for the randomized scores. Only one of the pairs of random groups had a difference of means in the range from 2 to 3. You can conclude that the study group was likely responsible for the higher scores.

Selected Answers

Topic 12

Lesson 12-1

1. Knowing whether events are mutually exclusive or independent can help you choose appropriate formulas for calculating probabilities. If events *A* and *B* are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$. If *A* and *B* are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$. **3.** Because Deshawn is likely to talk to his friends while playing basketball, the two events are not mutually exclusive. So, he cannot find the probability of one or the other happening by simply adding the probabilities of the two events.
5. a. 0.25, 25%, or $\frac{1}{4}$ **b.** 0.75, 75%, or $\frac{3}{4}$
7. $\frac{1}{6}$, $0.1\overline{6}$, or $16.\overline{6}\,\%$ **9.** Answers may vary. Sample: One situation in which every outcome is equally likely and every outcome is mutually exclusive is rolling a standard number cube. If you add the probabilities for getting each side of a number cube, 1, 2, 3, 4, 5, or 6, the sum is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$. So, if you add all of the probabilities in an experiment where every outcome is both equally likely and mutually exclusive, the sum of the probabilities should be 1. **11.** No; two events *A* and *B* are independent if and only if $P(A) \cdot P(B) = P(A \text{ and } B)$ and $(0.10)(0.15) \neq 0.05$. **13.** no **15.** yes
17. 0.72 or 72% **19.** 0.28 or 28%
21. a. $\frac{25}{49}$, **b.** independent
23. a. $\frac{18}{25}$ **b.** $\frac{1}{100}$ **c.** $\frac{1}{2}$ **d.** $\frac{9}{100}$ **25.** 25%
27. Part A 36% **Part B** 64%
Part C Yes, because even though the chance of rain on any one day is 50% or less, it is more likely than not to rain at some point during her visit.

Lesson 12-2

1. For any two events *A* and *B*, find $P(B \mid A)$ and $P(B)$ and compare them. If they are equal, then events *A* and *B* are independent; if not, then *A* and *B* are dependent events.
3. By definition $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$. Division by 0 is undefined, so $P(A)$ cannot equal 0. **5.** Taylor divided $P(\text{red and blue})$ by $P(\text{blue})$ and found $P(\text{red} \mid \text{blue})$ instead of $P(\text{blue} \mid \text{red})$.
$P(\text{blue} \mid \text{red}) = \frac{0.05}{0.8} = 0.0625$
7. a. $P(B \mid A) = \frac{2}{9}$ **b.** $P(A \mid B) = \frac{1}{4}$
9. No; $P(B \text{ and } T) = \frac{1}{3}$, $P(T) = \frac{1}{2}$, and $P(B) = \frac{1}{2}$. For the events to be independent,
$P(B \mid T) = \frac{P(B \text{ and } T)}{P(T)} = P(B)$ has to be true.
But $\frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \neq \frac{1}{2}$,
so the events are not independent.
11. The events are not independent, so $P(\text{red second} \mid \text{red first})$ is not the same as $P(\text{red first})$.
$P(\text{red second and red first})$
$= P(\text{red first}) \cdot P(\text{red second} \mid \text{red first})$
$= \frac{3}{10} \cdot \frac{2}{9} \approx 0.067$. **13. a.** $\frac{2}{9}$;
$P(\text{left crate} \mid \text{vegetable}) =$
$\frac{P(\text{left crate and vegetable})}{P(\text{vegetable})} = \frac{\frac{2}{20}}{\frac{9}{20}} = \frac{2}{9}$

Selected Answers

Topic 12

b. $\frac{8}{13}$; $P(\text{left crate} \mid \text{vegetable}) =$

$\frac{P(\text{left crate and vegetable})}{P(\text{vegetable})} = \frac{\frac{8}{20}}{\frac{13}{20}} = \frac{8}{13}$

15. about 0.52 or 52% **17.** 0.45 or 45% **19.** 0.08 or 8% **21.** dependent **23.** 0.225 or 22.5% **25.** $\frac{1}{12}$ or about 0.08 or 8% **27.** A, D

29. Part A

Student Exercise Survey

	Exercises Daily	Does Not Exercise Daily	Total
Male	28	22	50
Female	32	18	50
Total	60	40	100

Part B $P(\text{female} \mid \text{exercise}) = \frac{8}{15} \approx 0.53$; $P(\text{male} \mid \text{exercise}) = \frac{7}{15} \approx 0.47$

Part C Answers may vary. Sample: Of the students surveyed, 64% of females and 56% of males exercise daily. This suggests that females are somewhat more likely to exercise regularly than males.

Lesson 12-3

1. They provide formulas for finding the total number of outcomes needed to compute probabilities. **3.** There are more permutations of *n* items taken *r* at a time than there are combinations because the order of the items distinguishes between multiple permutations that contain the same items. **5.** No; the compound fraction is simplified incorrectly. 2! should appear in the numerator instead of the denominator, and the correct answer is $\frac{1}{10}$. **7.** combination

9. 1320 **11.** 70 **13.** $\frac{28}{91}$ **15.** $\frac{28}{91}$

17. 24,360; the arrangements are permutations because locks are specific, and if you enter the numbers in the wrong order, locks will not open.

19. The order is important in this problem. The number of ways to select 1 and 6 to form 16 is given by $_6P_2$, not $_6C_2$; $P(16) = \frac{1}{_6P_2} = \frac{1}{30}$.

21. 12; because the people are sitting around a circular table, the first position is determined and the order of the remaining 11 people is 11!.

23. permutation; 3,628,800

25. permutation; 40,320

27. combination(s); 630

29. a. Use a permutation because order matters. $P(DEB) = \frac{1}{_{10}P_3} = \frac{1}{720}$

b. Use combinations because order does not matter.

$P(\text{all vowels}) = \frac{_3C_3}{_{10}C_3} = \frac{1}{120}$

31. $\frac{1}{114}$; there are $_5C_3$ ways to select the tickets he wants and $_{20}C_3$ ways to select 3 tickets. So

$P(\text{3 winning baseball cap tickets})$

$= \frac{_5C_3}{_{20}C_3} = \frac{10}{1,140} = \frac{1}{114}$.

33. a. $\frac{_6P_6}{_{20}P_6} = \frac{1}{38,760}$

b. $\frac{1}{_{20}P_6} = \frac{1}{27,907,200}$ **35.** D

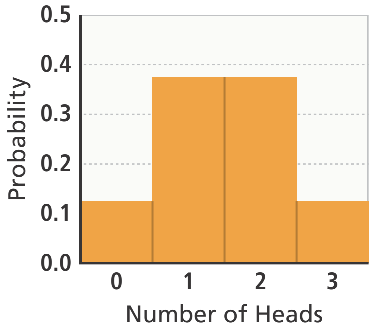
Selected Answers

Topic 12

Lesson 12-4

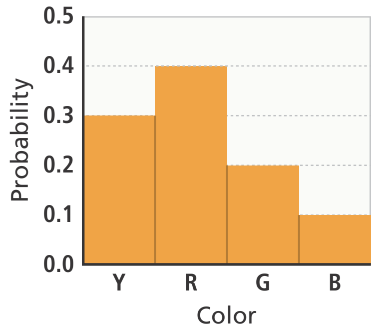
1. A probability distribution for an experiment shows how probabilities are assigned to each outcome of a sample space for the experiment. It can show whether the probability distribution is a uniform probability distribution or not, and if not, which outcomes are more likely or less likely. **3.** Rochelle rounded each probability to the nearest tenth. The sum of the probabilities should be 1, not 1.2. A correct distribution is P such that $P(A) = 0.25$, $P(B) = 0.25$, $P(C) = 0.25$, and $P(D) = 0.25$.

5.

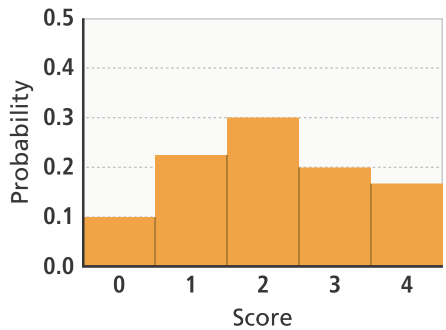


7. 13% **9.** 20% **11.** When you flip a fair coin, the probability of heads and the probability of tails are each $\frac{1}{2}$.
13. No; the results of the two selections are not independent. The teacher will not choose the same student twice, therefore the second selection is being made from fewer students, so the probability of success is different.
15. a. Sample: There are 6 ways to select 2 green marbles in 4 trials: GGRR, GRGR, GRRG, RGGR, RGRG, RRGG. That is, there are ${}_4C_2 = 6$ ways to choose 2 things out of 4. **b.** ${}_5C_3 = {}_5C_2$; choosing 3 out of 5 items is the same as not choosing those 3 items from the 5 items, or choosing 2 out of the remaining 5 items.

17.

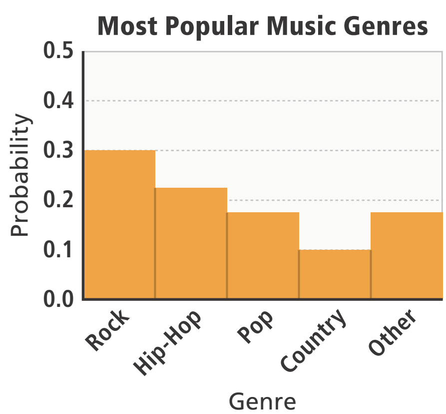


19.



21. No; the results of the two selections are not independent. Since the first card is not returned, the sample space is smaller for the next selection, so the probability changes. **23.** 5.6%
25. 28.2% **27. a.** Let P be the function, defined on the set {Rock, Hip-Hop, Pop, Country, Other} such that $P(\text{Rock}) = 30\%$, $P(\text{Hip-Hop}) = 22\%$, $P(\text{Pop}) = 19\%$, $P(\text{Country}) = 10\%$, $P(\text{Other}) = 19\%$.

b.



c. rock **29.** 1.7% **31.** D

Selected Answers

Topic 12

Lesson 12-5

1. Expected value provides a way to create a mean value for a variable that follows a probability distribution.
3. Yes; the carnival will lose an average of \$1.12 − \$1 = \$0.12 each time the game is played. The expected payout must be less than \$1 for the carnival to earn money.
5. when there are equal numbers of each kind of item. **7.** 7
9. a. \$960 **b.** no; Expected value tells you the average cost if you were booking many such flights.
11. The student incorrectly calculated the probabilities for each outcome. The area of the entire dartboard is 49π and the area of the inner circle is 4π. So, the area of the outer region is $49\pi - 4\pi = 45\pi$.
Therefore, the probability that a dart lands in the inner circle is $\frac{4\pi}{49\pi} = \frac{4}{49}$, the probability that it lands in the outer region is $\frac{45\pi}{49\pi} = \frac{45}{49}$, and the expected value is $\frac{4}{49}(4) + \frac{45}{49}(1) = \frac{61}{49} \approx 1.24$.
13. a. The cost of the policy with no deductible is \$600, so the policy with the \$1,000 deductible costs less.
b. Despite the fact that the policy with a deductible is less expensive, the consumer might choose the policy with no deductible to avoid an unexpected expense of \$1,000 sometime during the year. **15.** \$0.04 per dozen
17. \$76.89 **19.** 328.5 days

21. In the next year, if the company makes no changes, about 0.01 • 30,000 = 300 panels will be defective, incurring a cost of 300 • \$600 = \$180,000. If they make the manufacturing changes, the expected total cost is \$200,000 + 0.002 • 30,000 • \$600 = \$236,000. So, for the next year, it makes sense to not improve the process. But the company also predicts that sales will increase by 5,000 per year. Yearly costs of defective panels:

		Cost (in \$1000s)	
Year	Sales	With No Change	With Improved Process
1	30,000	\$180	\$236
2	35,000	\$210	\$242
3	40,000	\$240	\$248
4	45,000	\$270	\$254
5	50,000	\$300	\$260
6	55,000	\$330	\$266
7	60,000	\$360	\$272
8	65,000	\$390	\$278
9	70,000	\$420	\$284
10	75,000	\$450	\$290

For 3 years it is less costly to make no change. After 3 years, improving the process becomes a better option.
23. No warranty, because the expected cost for 3 years is less with no extended warranty than with a 2-year or 3-year warranty. Expected costs for 3 years:
No extended warranty:
\$599 + \$278(0.05) + \$278(0.08) = \$635.14
2-year extended warranty: \$599 + \$55 + \$278(0.08) = \$676.24 3-year extended warranty: \$599 + \$80 = \$679.00

Selected Answers

Topic 12

25. 14; 1 **27. Part A** Each side of the center square should be $\frac{1}{3}$ the side length of the large square. The ratio of the areas of the squares is 1:9, so the two regions have areas in the ratio 1:8.
Part B 3 points; Let x be the point value in the outer region. The probability of a chip landing in the center square is $\frac{1}{9}$ while the probability of landing in the outer region is $\frac{8}{9}$. Since a chip in the center square is worth 20 points, the expected value is $\frac{1}{9}(20) + \frac{8}{9}(x)$. For an expected value close to 5, solve this equation.

$$\frac{1}{9}(20) + \frac{8}{9}(x) = 5$$
$$\frac{20}{9} + \frac{8x}{9} = 5$$
$$20 + 8x = 45$$
$$8x = 25$$
$$x = 3.125$$

So, making the outer region worth 3 points solves the problem.
Part C The expected score is increased but not necessarily doubled. The amount of increase in the expected score depends on the point value of landing in each area.

Lesson 12-6

1. The probability of an outcome multiplied by its value can help you decide whether a game is fair, find the costs and benefits for a risky business venture, and make rational decisions based on expected values and returns rather than guesses. **3.** No; how the numbers are assigned or what meaning is attributed to them is a separate factor that determines whether a game is fair.

5. Answers may vary. Sample: Inspecting the graph of a probability distribution or a tree diagram makes it easier to see how likely or unlikely certain outcomes are, and inform your decisions. **7.** student 9, then student 4 **9.** Yes; they each have a $\frac{1}{6}$ chance of winning. **11.** Answers may vary. Sample: **a.** Roll your dice. If the product is even you get one point, if the product is odd you lose two points.
b. Roll your dice. If the sum is a prime number you get two points, if the sum is composite you lose one point.
13. The error is in the exponents for the probability terms. The correct formula is
$(12)(0.80)^{11}(0.20)^{1} + (1)(0.80)^{12}(0.20)^{0}$.
The correct probability is 27%. **15.** Fair; even numbers have the same number of chances, {2, 4, 6}, as odd numbers {1, 3, 5}. **17.** $33,875 **19. a.** about 0.00025, or 0.025% **b.** No; the probability is 0.025% that 2 or more people selected at random from a group of 5 people with an AGI between $50,000 and $75,000 will be audited. However, 2 out of 5 people in that tax bracket whose returns were prepared by ABC Tax Guys were audited, which is 40%. So I would not recommend ABC Tax Guys to a friend. **21. a.** 40% (rounded to the nearest whole percent) **b.** Based on the test of 50 new components, the experimental probability for failure is 2%. However, 50 is a relatively small sample, so I think that it is not reasonable to conclude that the new components have a lower failure rate than 4%. **c.** Further testing of the new parts is recommended since 50 is a relatively small sample. **23.** E

Selected Answers

Topic 12

Topic Review

1. Check students' work. See *Teacher's Edition* for details. **3.** mutually exclusive events **5.** complement **7.** not mutually exclusive **9.** not mutually exclusive **11.** 0.58 **13.** 0.40 **15.** dependent events **17.** 0.0075 or 0.75% **19.** permutation; 720 **21.** Permutations and combinations are ways to count an arrangement of items. The arrangements cannot occur in a fractional number of ways. **23.** 6.73% **25.** 0.30% **27.** 423.75

29. The player's calculation is correct: 3 points times 0% is 0. However, the player is basing the expected value on very few trials, so the value is likely to be inaccurate. If the player takes 20, 50, or 100 shots and bases her expected value calculation on the number of those shots that are successful, her estimate is likely to be better.